Discrete and Computational Geometry, WS1415
Exercise Sheet "1": Randomized Algorithms for Geometric Structures I
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Tuesday 21th of October, 14:00 pm**. *There will be a letterbox in the LBH building.*
- *You may work in groups of at most two participants.*
- *Please contact Hilko Delonge, hilko.delonge@uni-bonn.de, if you want to participate and have not yet signed up for one of the exercise groups.*
- *If you are not yet subscribed to the mailing list, please do so at https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom*

Exercise 1: Probability Space (4 Points)

Consider a standard 52-card deck of poker cards. Assume we do not distinguish cards by their suits, i.e., cards with the same number are identical. We want to select 5 cards from the deck. Please define the probability space as follows.

1. Please describe the sample space Ω. (The outcomes can be classified into 6 categories, and each category has different number of elements.)
2. Please describe the family $\mathcal{F}$ of events, e.g., the total number.
3. Please describe the probability function Pr by illustrating the probability for the elements in the sample space. (just one element for each category)
4. Let X be the random variable representing the sum of 5 cards. Please compute the expectation of X.

Exercise 2: Average Complexity of Sorting (4 Points)

Given a set N of n real numbers, please analyze the average complexity for the following sorting algorithms over all the $n!$ permutation sequences of N.

- Insertion Sort
- Merge Sort
- Quick Sort (always select the first element)

Exercise 3: Vertical Trapezoidal Decomposition (4 Points)

Given a set N of n line segments with a total number k of intersection in the plane, let $S_1, S_2, \ldots, S_n$ be a random sequence of N, and let N^i be $\{S_1, S_2, \ldots, S_i\}$. **General Position Assumption: No two endpoints in N share the same x-coordinate**. Please prove the following.

1. The vertical trapezoidal decomposition $H(N)$ of N has $O(n+k)$ trapezoids (faces) even if more than two line segments can intersects at the same point.
2. The expected number of trapezoids in $H(N^i)$ is $O(i + ki^2/n^2)$. (Hint: the expected number of intersections)