- *Written solutions have to be prepared until* **Tuesday 16th of December 14:00 pm**. *There will be a letterbox in the LBH building.*

- *You may work in groups of at most two participants.*

- *Please contact Hilko Delonge, hilko.delonge@uni-bonn.de, if you want to participate and have not yet signed up for one of the exercise groups.*

- *If you are not yet subscribed to the mailing list, please do so at https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom*

Exercise 22: **Randomized Incremental Algorithm for Abstract Voronoi Diagrams (History Graph)** **(4 Points)**

Consider an admissible bisecting curve system $(S, \mathcal{J})$, and make a general position assumption that no four curves in $\mathcal{J}$ intersect at the same point. Let $s_1, s_2, \ldots, s_n$ be a random sequence of S, and let R^i be $\{\infty, s_1, s_2, \ldots, s_i\}$. Please develop a randomized algorithm to construct the abstract Voronoi diagram $V(S)$ by computing $V(R^2), V(R^3), \ldots, V(R^n)$ iteratively using the history graph. In other words, for $i \geq 2$, obtain $V(R^{i+1})$ from $V(R^i)$ by insertion s_{i+1}. Let a configuration be a Voronoi edge of $V(R^i)$, for $2 \leq i \leq n$

1. Define the parent and child relation between a configuration in $V(R^i) \setminus V(R^{i+1})$ and a configuration in $V(R^{i+1}) \setminus V(R^i)$

2. Please prove that if a site conflicts a configuration, there exists a path from the root of the history graph to the configuration along which all configuration is in conflict with the site.

3. Prove that the expected time complexity of inserting s^i is $O(\log i)$

Exercise 23:   Removal of General Position Assumption   (4 Points)

Consider an admissible bisecting curve system $(S, \mathcal{J})$ without the general position assumption that no four curves in $\mathcal{J}$ intersect at the same point. In other words, more than three curves in $\mathcal{J}$ can intersect at the same point, and the degree of a Voronoi vertex can be more than three. Please complete the following

- Use a constant number of sites to define a Voronoi edge, i.e., formulate a configuration for a Voronoi edge. Note that a site can appear more than once in a configuration.

- Please describe how to update the conflict graph after inserting s into $V(R)$.