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Randomized Algorithms and
Probabilistic Analysis
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Problem Set 7

The joint distribution function of two continuous random variables X and Y is defined by

$$F_{X,Y}(x,y) = \Pr(X \leq x, Y \leq y)$$

for all $x, y \in \mathbb{R}$. The joint density function is the function $f_{X,Y} : \mathbb{R}^2 \to \mathbb{R}_0^+$ that satisfies

$$F(X,Y)(x,y) = \Pr(X \leq x, Y \leq y) = \int_{-\infty}^{y} \int_{-\infty}^{x} f_{X,Y}(u,v)\,\mathrm{d}u\,\mathrm{d}v.$$

If X and Y are independent, then $\Pr(X \leq x, Y \leq y) = \Pr(X \leq x) \cdot \Pr(Y \leq y)$, so $F_{X,Y}(x,y) = F_X(x) \cdot F_Y(y)$ in this case. Similarly, $f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$ for independent X and Y and for all $x, y \in \mathbb{R}$. For any $A \subset \mathbb{R}^2$ (with suitable restrictions that ensure that the following integral can be computed),

$$\Pr((X,Y) \in A) = \int_A f_{X,Y}(x,y)\,\mathrm{d}x\,\mathrm{d}y.$$

Problem 1

Let X and Y be independent, uniform random variables on $[0,1]$. Find the density function and distribution function for $X + Y$.

Problem 2

We agree to try to meet between 12 and 1 for lunch at our favorite sandwich shop. Because of our busy schedules, neither of us is sure when we'll arrive; we assume that, for each of us, our arrival time is uniformly distributed over the hour. So that neither of us has to wait too long, we agree that we will each wait exactly 15 minutes for the other to arrive, and then leave. What is the probability we actually meet each other for lunch?