

Problem Set 6

Problem 1

Assume that we throw $m = \lfloor 5n \log_2 n \rfloor$ balls into n baskets. For each ball, the basket is chosen uniformly at random. Let X_i be the number of balls in basket i for $i \in \{1, \dots, n\}$, and set $X = \max_{i=1, \dots, n} X_i$. Use Thm 3.9 to show that

$$\Pr(X \geq 30 \log_2 n) \leq \frac{1}{n^c}$$

holds for a suitable constant $c > 0$.

Problem 2

Show that $\binom{n}{k} \in \Theta(n^k)$ if $k \in \mathbb{N}$ is a constant. Is this still true if $k = n/2$ (and thus not a constant?).