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# Problem Set 7

## Problem 1

Show that the recurrence

$$h_0 = 1 + h_1, \quad h_n = 0, \quad \forall j \in \{0, \ldots, n-1\} : h_j \leq 1 + \frac{2}{3}h_{j-1} + \frac{1}{3}h_{j+1}$$

implies that $h_j \leq 2^{n+2} - 2^{j+2} - 3(n-j)$ for all $j \in \{0, \ldots, n\}$. *Hint:* First show by induction that for all $j \in \{0, \ldots, n-1\}$, $h_j \leq h_{j+1} + 2^{j+2} - 3$ holds.

(This task completes the proof of Lemma 4.7).

## Problem 2

Let $a \in \{0,1\}^n$ be uniformly chosen from all possible assignments $\{0,1\}^n$, and let $a^*$ be an unknown but fixed assignment. Let $r := |\{i \in \{1, \ldots, n \mid a_i = a_i^*\}|$ be the number of positions where $a$ and $a^*$ agree. Show that

$$\Pr(r \geq n/2) \geq 1/2.$$