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Randomized Algorithms and
Probabilistic Analysis
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Problem Set 11

Problem 1

Consider an arbitrary binary optimization problem with linear objective $c^T x$ and solution set $\mathcal{S} \subseteq \{0,1\}^n$ as discussed in Chapter 7. Recall that the winner gap Δ is defined as

$$\Delta := cx^* - cx^{**}$$

where x^* is an arbitrary optimal solution and x^{**} is a solution that is optimal amongst all solutions in $\{x \in \mathcal{S} \mid x \neq x^*\}$. Find better upper bounds on $\Pr(\Delta \leq \epsilon)$ than the bound provided by Lemma 7.3 for the following scenarios:

1. The c_i are ϕ-perturbed numbers from $[0,1]$ (instead of $[-1,1]$).

   Show that $\Pr(\Delta \leq \epsilon) \leq n\phi\epsilon$.

2. The c_i are numbers from $[1,e]$ that are chosen independently from the distribution with the density

$$f(x) = \begin{cases} \frac{1}{x} & \text{for all } x \in [1,e] \\ 0 & \text{else.} \end{cases}$$

   Show that $\Pr(\Delta \leq \epsilon) \leq n \ln(1+2\epsilon)$.

Problem 2

Discuss how the SSP algorithm performs on the following input network. For each edge, the first number is the capacity, the second number is the cost. The value T is a some integer.