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Randomized Algorithms and
Probabilistic Analysis
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Problem Set 9

Problem 1

Suppose that we roll a standard fair dice seventeen times (independently). What is the probability that the sum is divisible by six? Use the principle of deferred decisions.

Problem 2

We want to get more familiar with the notion of ϕ-perturbed values. Assume that c is drawn independently from a distribution with density function $f(x)$, where

1. $f(x)$ is the density of the uniform distribution over the interval $[4, 4+u]$ for a constant $u > 0$.
2. $f(x) = \begin{cases} x^2 \cdot \frac{3}{u^3} & \text{for } x \in [0,u] \\ 0 & \text{else} \end{cases}$ for a constant $u \in (0, \infty)$.
3. $f(x) = \begin{cases} \frac{1}{x} \cdot \frac{1}{\ln u} & \text{for } x \in [1,u] \\ 0 & \text{else} \end{cases}$ for a constant $u \in (1, \infty)$.

For all three cases, do the following:

a) Compute (a best possible) ϕ such that $f(x) \leq \phi$ for all $x \in \mathbb{R}$.

b) Give a best possible upper bound $\nu(u, \epsilon)$ for the probability to draw a number from a given fixed interval of width $\epsilon \in (0, 1)$.

c) Assume someone tells us that c is 3-perturbed. Based on this fixed $\phi = 3$, for which of the three scenarios do we get the largest (i.e. worst) $\nu(u, \epsilon)$?

Problem 3

Let $G = (V, E)$ be a graph with edge lengths $\ell : E \to [0, 1]$ and edge costs $c : E \to [0, 1]$. Let $\{s, t\} \in V$. We want to find an s-t-path with minimum length as well as minimum total costs. In general there is no such path that optimizes both criteria simultaneously and we are interested in the set of Pareto-optimal paths. Give an algorithm to find the set of Pareto-optimal paths and analyze its worst-case and smoothed running time. In the analysis of the smoothed running time assume that the edge costs are ϕ-perturbed values.

Problem 4

Consider the same setting as in the previous problem, except that we are now interested in the TSP. That is, we assume that G is a complete graph and we want to find a Hamiltonian cycle that is as short and as cheap as possible. Can the set of Pareto-optimal Hamiltonian cycles be computed efficiently (i.e., in polynomial time with respect to the input size and the size of the Pareto set)?