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Algorithmic Game Theory and the Internet

Summer Term 2019

Exercise Set 6

Just this once, please hand in your solutions via email to matthias.buttkus@uni-bonn.de or at the Institute of Computer Science, Dept. V, Room 2.013.

Exercise 1: (2+1 Points)
Consider the following single-item auction with $n \geq 2$ bidders. The bidders simultaneously submit their bids $b_i \geq 0$. However, the item will always be allocated to the bidder with index 1 and the mechanism will make him/her pay the bid of the bidder with index 2.

(a) Show that the described mechanism is truthful.

(b) We call a mechanism to be *individual rational* if for all bidders $i \in \mathcal{N}$ bidding truthfully against an arbitrary bid profile of the other players never leads to a negative utility: If $v_i(x) \geq 0$ for all allocations $x \in X$, then $u_i((v_i, b_{-i}), v_i) \geq 0$.

Show that the given mechanism is not individual rational.

Exercise 2: (4 Points)
Analogous to the auctions that we defined in the lecture, we will consider the following *Third-Price Auction*. Just like in the first- and second-price auctions, bidders simultaneously submit their bids $b_i \geq 0$ and the winner will be determined as the bidder with the highest bid. Finally, the mechanism will make him/her pay the third highest bid. Prove that the described mechanism is not truthful.

Exercise 3: (2+3 Points)
Consider a second-price auction with a fixed value profile $(v_i)_{i \in N}$. Since the value profile is fixed, we get a normal-form utility-maximization game.

(a) Show that there exists a pure Nash equilibrium in the defined game.

(b) Now, consider a game in which only two players participate and $v_1 \gg v_2$ holds. Prove that even in this setting there exists a pure Nash equilibrium such that bidder 2 wins.

Exercise 4: (4+4 Points)
We consider an auction of k identical items. Each bidder can acquire at most one of the items. If bidder i gets one of the items, she has a value of v_i. Otherwise, that is, if she does not get an item, she has a value of 0.

(a) State a generalization of the second-price auction and prove that it is truthful (the second-price auction covers the case of $k = 1$). Follow steps in the spirit of Lecture 10.

(b) Now, consider a mechanism which sequentially performs k second-price auctions. That is, initially each bidder reports one bid. Then, in each auction, one item is sold among the remaining players using their initial bids. Show that truthful bidding does not necessarily lead to a pure Nash equilibrium even in the special case of three players and $k = 2$.