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April 29, 2020
Due: Mai 6, 2020 at noon

Algorithms and Uncertainty

Summer Term 2020

Exercise Set 2

Exercise 1: (5 Points)

As on the first problem set, given an instance of Set Cover, let $f = \max_{e \in U} |\{S \in \mathcal{S} \mid e \in S\}|$ denote the frequency of the set system.
Use our results from the third lecture to devise an online algorithm that is $O(\log f)$-competitive for fractional set cover and prove this. You may assume that f is known beforehand.

Hint: One bound in the analysis from the lecture can be improved for $f < n$. Use it to adapt the algorithm.

Exercise 2: (2+2 Points)

Consider the following rounding algorithm for the Online Set Cover problem. In step t, as a new element e arrives, holding a solution to the factional set cover problem, we pick all sets $S \in \mathcal{S}$ for which $x_S^{(t)} \geq 1/f$. As in the first exercise, let $f = \max_{e \in U} |\{S \in \mathcal{S} \mid e \in S\}|$ denote the frequency of the set system which is known beforehand.

(a) Show that the rounded integral solution is feasible for the Online Set Cover problem.

(b) Show that if we use an α-competitive algorithm for the fractional problem, the algorithm for the integral problem is αf-competitive.

Exercise 3: (5 Points)

Consider the following randomized rounding for fractional Ski Rental. In step t, flip an independent biased coin: With probability $\frac{x_{\text{buy}}^{(t)} - x_{\text{buy}}^{(t-1)}}{1 - x_{\text{buy}}^{(t-1)}}$ buy the skis, otherwise rent them. Show that if the underlying fractional algorithm is α-competitive so is the randomized integral algorithm.

Exercise 4: (3 Points)

The Online Bipartite Vertex Cover problem is defined as follows: We are given a bipartite graph with vertices $V = L \cup R$. The nodes in L are offline which means they are present at $t = 0$. Nodes in R are online and revealed one at a time together with its incident edges. Each vertex $v \in V$ has a cost c_v. We need to maintain a feasible vertex cover in every step with the goal of minimizing the overall incured costs.
Consider the ski rental problem in the simplified version, i.e. we assume that every day is a skiing day but we do not know the number of days in advance. Show in a constructive way that the ski rental problem is a special case of the Online Bipartite Vertex Cover problem.