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May 26, 2020
Due: June 3, 2020 at noon

Algorithms and Uncertainty

Summer Term 2020

Exercise Set 5

Lectures: *Due to the Dies Academicus on May 27, there will be no lecture on Wednesday. Further, there will also be no lectures on June 1 (Pfingstmontag) and June 3. On June 3, a Q&A session will be offered instead.*
Tutorials: *We will discuss this sheet on June 4 in a quick tutorial session. The week after, there is a public holiday (Fronleichnam) on Thursday, June 11, on which we will have no tutorials. Problem Set 6 will only be available in the week after the holidays, i.e. on June 10.*

Exercise 1: (3+4 Points)

We extend the problem of opening boxes from Lecture 10. We are still allowed to open k boxes, but now, we may keep up to ℓ prizes instead of only one.

(a) Derive a linear program such that the expected reward of any adaptive policy is upper-bounded by the value of the optimal LP solution. Give a proof.

(b) Show that the adaptivity gap is still at most 8.

Exercise 2: (4 Points)

Show that Stochastic Set Cover can be reduced to the deterministic problem. To this end, define a different universe of elements U', family of subsets $\mathcal{S}'$, and costs $(c'_{S'})_{S' \in \mathcal{S}'}$ appropriately. Any solution of this Set Cover instance then corresponds to a policy of the same cost.

Exercise 3: (3+3+3 Points)

We consider the Stochastic Vertex Cover problem. The edge set $A \subseteq E$ is uncertain, but drawn from a known probability distribution. The probability that the edge set is $A \subseteq E$ is given by p_A. Our goal is to compute a Vertex Cover of mimimum cost for the graph $G = (V, A)$. Before A is revealed, we have to pay c^{I}_v for v, afterwards $c^{\mathrm{II}}_v \geq c^{\mathrm{I}}_v$.

(a) Derive an LP such that every policy corresponds to a feasible solution. Consider variables x_v denoting if v is picked in the first stage and $y_{A,v}$ if the edge set is A and v is picked in the second stage.

In order to compute a feasible policy, we use the following algorithm which uses an optimal LP solution (x^*, y^*):

1. In the first stage, pick all vertices for which $x^*_v \geq \frac{1}{4}$.
2. In the second stage, when knowing A, pick all vertices for which $y^*_{A,v} \geq \frac{1}{4}$.

(b) Show that this algorithm always computes a feasible policy.

(c) Show that the expected cost of the computed policy is at most 4-times the expected cost of the optimal policy.