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June 10, 2020
Due: June 17, 2020 at noon

Algorithms and Uncertainty

Summer Term 2020

Exercise Set 6

Exercise 1: (3+4+2 Points)

We consider the following modified version of the Boosted Sampling algorithm for Stochastic Steiner Tree from the lecture. The only difference is that it uses ℓ sets $S_1, \ldots, S_\ell$ in the first phase. Show that the approximation guarantee is $\max\{2(1 + \frac{\lambda}{\ell+1}), 2(\frac{\ell}{\lambda} + 1)\}$. To this end, consider the following tasks concerning the cost of the respective phases.

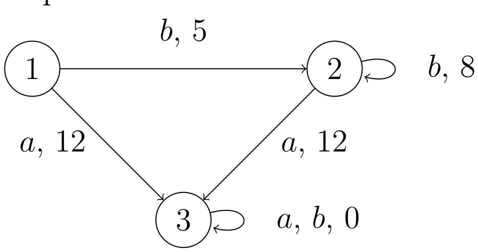
(a) Give an appropriate analysis for the first phase.

(b) Give an appropriate analysis for the second phase.

(c) Combine both results to derive the desired approximation guarantee.

Exercise 2: (2+2+2 Points)

We consider a Markov decision process with $\mathcal{S} = \{1, 2, 3\}$, $\mathcal{A} = \{a, b\}$. The state transitions are deterministic as displayed in this diagram; the numbers in the edge labels are the respective rewards.



We consider an infinite time horizon with discount factor $\gamma = \frac{1}{2}$.

(a) Give an optimal policy and the function $s \mapsto V^*(s)$.

(b) Perform the first six steps of value iteration starting from $W^{(0)} = (0, 0, 0)$.

(c) Perform policy iteration until convergence starting from the policy that always uses action a.

Exercise 3: (4 Points)

We define a more cautious version of value iteration. It uses the operator T', which is defined by $T'(W) = \eta T(W) + (1 - \eta)W$ for an arbitrary $\eta \in (0, 1)$. Show that this algorithm also converges to the unique fixed point of T.