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Due: July 1, 2020 at noon

Algorithms and Uncertainty

Summer Term 2020

Exercise Set 8

Exercise 1: (3+2 Points)

Recall the regret definition from the lecture: $\text{Regret}^{(T)} = L_{\text{Alg}}^{(T)} - \min_i \sum_{t=1}^{T} \ell_i^{(t)}$.
We want to understand the order of minimum and sum in the second term. Therefore, work on the following tasks.

(a) Use Yao's principle to show that for every (randomized) algorithm there is a sequence $\ell^{(1)}, \ldots, \ell^{(T)}$ such that $L_{\text{Alg}}^{(T)} \geq \left(1 - \frac{1}{n}\right) T$ but $\sum_{t=1}^{T} \min_i \ell_i^{(t)} = 0$.

(b) Discuss the importance of the order of sum and minimum in the regret definition using your results from (a).

Exercise 2: (4 Points)

We consider a generalization of the algorithm *Weighted Majority* for classifiers with k different classes. (The case covered in the lecture, binary classification, is $k = 2$.) In each step, the algorithm chooses a class, which is recommended by the largest number of classifiers (so the class has a plurality).
Show that this algorithm makes at most $(2 + 2\eta) \min m_i + 2 \ln n / \eta$ errors, where m_i is the number of errors of classifier i.

Exercise 3: (4 Points)

Consider the modified update rule for Multiplicative Weights that sets $w_i^{(t+1)} = w_i^{(t)} \cdot (1 - \ell_i^{(t)} \eta)$.
Show that Theorem 16.3 still holds.

Exercise 4: (4 Points)

Show that every no-regret algorithm in the experts setting has to be randomized. Consider the case $n = 2$ and for every deterministic algorithm construct a sequence such that $L_{\text{Alg}}^{(T)} = T$ and $\min_i L_i^{(T)} \leq \frac{T}{2}$.