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April 17, 2023
Due date: April 25 , 2023 at 10:00 p.m.

Algorithmic Game Theory

Summer Term 2023

Exercise Set 3

If you want to hand in your solutions for this problem set, please send them via email to anna.heuser@uni-bonn.de *by Tuesday evening – make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to anna.heuser@uni-bonn.de *containing the* ***task*** *which you would like to present and in* ***which of the tutorials*** *you would like to do so. Deadline for the email is Tuesday, 10:00 pm. Please note that the tasks will be alloecated via a first-come-first-served procedure, so sending this email earlier than Tuesday evening is highly recommended.*

Exercise 1: (3 Points)

Have a look at the proof of Nash's Theorem (4.3) in which normal-form payoff-maximization games are considered. Let $\mathcal{N} = \{1, \ldots, n\}$ and $S_i = \{1, \ldots, m_i\}$ for all $i \in \mathcal{N}$. The set of mixed states X can be considered as a subset of $\mathbb{R}^m$ with $m = \sum_{i=1}^n m_i$.

Show that X is convex and compact.

Exercise 2: (3+4 Points)

Consider the local search problem *Positive Not-All-Equal kSat* (Pos-NAE-kSAT) which is defined the following way:

Instances: Propositional logic formula with n binary variables $x_1, \ldots, x_n$ that is described by m clauses $c_1, \ldots, c_m$. Each clause c_i has a weight w_i and consists of exactly k literals, which are all positive (i.e., the formula does not contain any negated variable $\overline{x}_i$).

Feasible solutions: Any variable assignment $s \in \{0, 1\}^n$

Objective function: Sum of weights of clauses c_i in which not all literals are mapped to the same value.

Neighbourhood: Assignments s and s' are *neighbouring* if they differ in the assignment of a single variable.

You can assume that Pos-NAE-kSAT is in PLS. Now:

(a) Show that Pos-NAE-2SAT $\leq_{PLS}$ MaxCut

(b) Show that Pos-NAE-3SAT $\leq_{PLS}$ Pos-NAE-2SAT

Exercise 3: (4 Points)

We define a Congestion Game to be *symmetric*, if $\Sigma_1 = \ldots = \Sigma_n$. Let $PNE_{\text{Cong. Game}}$ and $PNE_{\text{Sym. Cong. Game}}$ be the local search problems in PLS of finding a pure Nash equilibrium of a general or symmetric Congestion Games, respectively.

Show: $PNE_{\text{Cong. Game}} \leq_{PLS} PNE_{\text{Sym. Cong. Game}}$.

Hint: Add an auxiliary resource for each player with a suitable delay function.