

Algorithmic Game Theory

Summer Term 2023

Exercise Set 9

If you want to hand in your solutions for this problem set, please send them via email to anna.heuser@uni-bonn.de by Tuesday evening – make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.

If you would like to present one of the solutions in class, please also send an email to anna.heuser@uni-bonn.de containing the **task** which you would like to present and in **which of the tutorials** you would like to do so. Deadline for the email is Tuesday, 10:00 pm. Please note that the tasks will be allocated via a first-come-first-served procedure, so sending this email earlier than Tuesday evening is highly recommended.

Exercise 1: (6 Points)

We work in the setup of combinatorial auctions with m (possibly heterogeneous) items M . Bidders report bids for items and afterwards each item is sold in a separate second-price auction (item bidding). Prove the following theorem.

Theorem. Consider a pure Nash equilibrium b of item bidding with second-price payments and unit-demand bidders. Let X_1, \dots, X_n be the resulting allocation. If for all bidders i we have $\sum_{j \in X_i} b_{i,j} \leq v_i(X_i)$ (weak no-overbidding), then $\sum_{i \in N} v_i(X_i) \geq \frac{1}{2}OPT(v)$.

Hint: Make use of the following deviation bids: Consider the welfare-maximizing allocation on v . Let j_i be the item that is assigned to bidder i in this allocation. If i does not get any item, set j_i to \perp . Set $b_{i,j}^* = v_{i,j}$ if $j = j_i$ and 0 otherwise. Now, derive a proof in the spirit of Theorem 17.2.

Exercise 2: (4 Points)

Consider m items and n bidders. We define a generalization of Walrasian equilibria: Let $S = (S_1, \dots, S_n)$ be an allocation of items to bidders and $q \in \mathbb{R}_{\geq 0}^m$ be a price vector. We call the pair (q, S) an ϵ -approximate Walrasian equilibrium if unallocated items have price 0, every bidder i has non-negative utility $v_i(S_i) - \sum_{j \in S_i} q_j \geq 0$, and every bidder receives items within ϵ of its favorite bundle, i.e., $v_i(S_i) - \sum_{j \in S_i} q_j \geq v_i(S'_i) - \sum_{j \in S'_i} q_j - \epsilon$ for every bundle S'_i .

Prove an approximate version of the First Welfare Theorem: If (q, S) is an ϵ -approximate Walrasian equilibrium, then the social welfare of an optimal allocation S^* cannot surpass the one of S by more than $\min\{m, n\} \cdot \epsilon$.

Exercise 3: (4 Points)

Recall the valuation functions of single-minded bidders from Definition 12.2. Let the maximum bundle size be defined by $d = \max_{i \in N} |S_i^*|$. Show that in the case of single-minded bidders with maximum bundle size d , item bidding with first price payments is $(\frac{1}{2}, 2d)$ -smooth.

Hint: In order to define deviation bids $b_{i,j}^*$, consider a welfare-maximization allocation on v . If bidder i does not get his bundle in the optimal allocation, then define $b_{i,j}^* = 0$ for all items $j \in M$. Otherwise, define $b_{i,j}^* = \frac{v_i}{2d}$ for all $j \in S_i^*$ and $b_{i,j}^* = 0$ if $j \notin S_i^*$. That is, each winner in the optimal allocation equally divides the value for his bundle among all items of the bundle and bids half of it.