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Due date: May 14, 2024 at 10:00 p.m.

Algorithmic Game Theory

Summer Term 2024

Exercise Set 3

If you want to hand in your solutions for this problem set, please send them via email to anna.heuser@uni-bonn.de *by Tuesday evening – make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to anna.heuser@uni-bonn.de *containing the* ***task*** *which you would like to present and in* ***which of the tutorials*** *you would like to do so. Deadline for the email is Tuesday, 10:00 pm. Please note that the tasks will be alloecated via a first-come-first-served procedure, so sending this email earlier than Tuesday evening is highly recommended.*

Exercise 1: (3+4 Points)

Consider the local search problem *Positive Not-All-Equal kSat* (Pos-NAE-kSAT) which is defined the following way:

Instances: Propositional logic formula with n binary variables $x_1, \ldots, x_n$ that is described by m clauses $c_1, \ldots, c_m$. Each clause c_i has a weight w_i and consists of exactly k literals, which are all positive (i.e., the formula does not contain any negated variable $\overline{x}_i$).

Feasible solutions: Any variable assignment $s \in \{0, 1\}^n$

Objective function: Sum of weights of clauses c_i in which not all literals are mapped to the same value.

Neighbourhood: Assignments s and s' are *neighbouring* if they differ in the assignment of a single variable.

You can assume that Pos-NAE-kSAT is in PLS. Now:

(a) Show that Pos-NAE-2SAT $\leq_{PLS}$ MaxCut

(b) Show that Pos-NAE-3SAT $\leq_{PLS}$ Pos-NAE-2SAT

Exercise 2: (4 Points)

We define a Congestion Game to be *symmetric*, if $\Sigma_1 = \ldots = \Sigma_n$. Let $PNE_{\text{Cong. Game}}$ and $PNE_{\text{Sym. Cong. Game}}$ be the local search problems in PLS of finding a pure Nash equilibrium of a general or symmetric Congestion Games, respectively.

Show: $PNE_{\text{Cong. Game}} \leq_{PLS} PNE_{\text{Sym. Cong. Game}}$.

Hint: Add an auxiliary resource for each player with a suitable delay function.

Exercise 3: (3+3 Points)

We want to derive properties of the sets of correlated and coarse correlated equilibria.

(a) Show that the set of correlated equilibria of a cost-minimization game Γ is convex, i.e. for two correlated equilibria p, p' and $\lambda \in [0, 1]$, also $\lambda p + (1 - \lambda)p'$ is a correlated equilibrium.

(b) Show that every correlated equilibrium is also a coarse correlated equilibrium.