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June 10, 2024
Due date: June 18, 2024 at 10:00 p.m.

Algorithmic Game Theory

Summer Term 2024

Exercise Set 6

If you want to hand in your solutions for this problem set, please send them via email to anna.heuser@uni-bonn.de *by Tuesday evening – make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to anna.heuser@uni-bonn.de *containing the* ***task*** *which you would like to present and in* ***which of the tutorials*** *you would like to do so. Deadline for the email is Tuesday, 10:00 pm. Please note that the tasks will be alloecated via a first-come-first-served procedure, so sending this email earlier than Tuesday evening is highly recommended.*

Exercise 1: (4 Points)

An *all-pay auction* is a single-item auction defined in almost the same manner as a first-price auction: Each bidder reports a bid $b_i \geq 0$. The bidder with the highest bid wins the item. However, every bidder must pay their own bid regardless of whether they win the item or not.

Be inspired by the steps of Section 2 in the notes of Lecture 15 to derive the symmetric Bayes-Nash equilibrium of an all-pay auction with n bidders and identical distributions.

Exercise 2: (4 Points)

Show that if a mechanism is (λ, μ)-smooth and players have the possibility to withdraw from the mechanism then $PoA_{\mathsf{CCE}} \leq \frac{\max\{1,\mu\}}{\lambda}$.

Exercise 3: (3 Points)

Recall the auction of k identical items. Bidder i has value v_i if he/she gets one of the items, 0 otherwise.
We define a mechanism as follows: the bidders who reported the k highest bids win an item. Each of them has to pay their respective bids.
Show that if losers (i.e. bidders who do not get any item) pay their bids, this mechanism is $(\frac{1}{2}, 2)$-smooth.