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June 24, 2024
Due date: July 2, 2024 at 10:00 p.m.

Algorithmic Game Theory

Summer Term 2024

Exercise Set 8

If you want to hand in your solutions for this problem set, please send them via email to anna.heuser@uni-bonn.de *by Tuesday evening – make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to anna.heuser@uni-bonn.de *containing the **task** which you would like to present and in **which of the tutorials** you would like to do so. Deadline for the email is Tuesday, 10:00 pm. Please note that the tasks will be alloecated via a first-come-first-served procedure, so sending this email earlier than Tuesday evening is highly recommended.*

Exercise 1: (2+2+2 Points)

Determine the virtual value function φ of the following probability distributions.

(a) Uniform distribution on the interval $[a, b]$.

(b) Exponential distribution with rate $\lambda > 0$ (defined on $[0, \infty)$).

(c) The distribution given by the cumulative distribution function $F(v) = 1 - \frac{1}{(v+1)^c}$ defined on the interval $[0, \infty)$, where $c > 0$ is considered to be an arbitrary constant.

Which of the stated distributions are regular?

Exercise 2: (1+3 Points)

Once again, consider a single-item auction with two bidders whose valuations are drawn independently from a uniform distribution over $[0, 1]$.

(a) Prove that the random variables $\varphi_i(v_i)$ are distributed according to a uniform distribution on $[-1, 1]$.

(b) Define a second-price auction with *reserve price* p. Let v_1 and v_2 be the valuations of the bidders. The allocation and payment rule will be determined according to the following cases:

1. $\min\{v_1, v_2\} \geq p$: Like in the second price auction.
2. $\max\{v_1, v_2\} < p$: Nobody gets the item and no payments.
3. $v_1 \geq p > v_2$: Bidder 1 gets the item and has to pay p.
4. $v_2 \geq p > v_1$: Analogous to 3.

Utilize subtask (a) and the results of the lecture in order to determine the expected revenue of a second-price auction with reserve price $p \in [0, 1]$.

Exercise 3: (3+3 Points)

We want to discuss non-truthful mechanisms. Therefore, consider a single-item first-price auction with n bidders whose values are drawn uniformly at random from $[0, 1]$.

(a) Show that each bidder reporting a $\frac{n-1}{n}$-fraction of their actual value is a Bayes-Nash equilibrium.

(b) Compute the expected revenue of the first-price auction at equilibrium.