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July 8, 2024
Due date: July 16, 2024 at 10:00 p.m.

Algorithmic Game Theory

Summer Term 2024

Exercise Set 10

If you want to hand in your solutions for this problem set, please send them via email to anna.heuser@uni-bonn.de *by Tuesday evening – make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to anna.heuser@uni-bonn.de *containing the* ***task*** *which you would like to present and in* ***which of the tutorials*** *you would like to do so. Deadline for the email is Tuesday, 10:00 pm. Please note that the tasks will be alloecated via a first-come-first-served procedure, so sending this email earlier than Tuesday evening is highly recommended.*

Exercise 1: (3 Points)

Prove that the men-proposal algorithm is not DSIC for the right-hand side (the women). For this purpose, give an instance of the stable matching problem in which, by lying about her preferences during the execution of the men-proposing algorithm, a woman can end up with a man that she prefers over the man she would have ended up with had she told the truth.

Exercise 2: (4 Points)

Show that there are instances of the stable matching problem in which the Gale-Shapley Algorithm (men-proposing algorithm) runs for $\Omega(n^2)$ iterations before terminating (with a stable matching). For this purpose, state an instance of the problem depending on n with suitable chosen preference orders and lower bound the number of iterations of the algorithm.

Hint: Consider an instance with $|U| = |V| = n$. Try to enforce exactly one rejection per iteration.

Exercise 3: (3 Points)

Recall the setting for Cake Cutting from Lecture 24. Show that if valuations are identical, i.e. $v_i(.) = v_j(.)$ for all $i, j \in N$, then the notions of Proportionality, Envy-Freeness and Equitability coincide.

Exercise 4: (3 Points)

Consider the algorithm (which is also known as the *moving-knife algorithm*) given in Section 4 of Lecture 24 that determines a proportional allocation for any number of agents n.

Show that even in the case of three agents the allocation of the algorithm might not be envy-free.