

## Algorithmic Game Theory

Summer Term 2026

### Exercise Set 1

If you would like to submit your solutions for this problem set, please send them via email to [aheuser1@uni-bonn.de](mailto:aheuser1@uni-bonn.de) by Monday evening. Submitting solutions in groups is also possible.

If you would like to present one of your solutions in class, please use the following link to book a presentation slot by Monday evening:

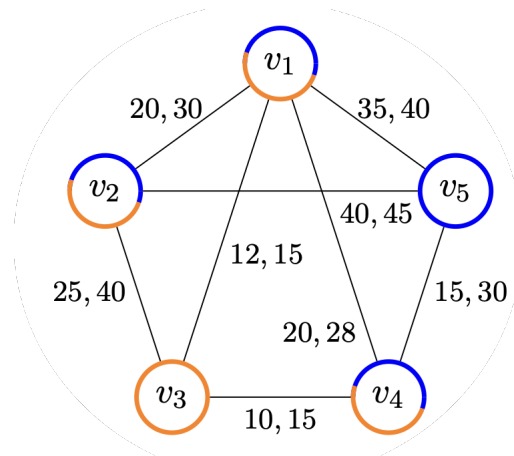
<https://terminplaner6.dfn.de/b/811054ac6e22f373eaec643615b1a9d2-1699817>

A short meeting to discuss your solution is mandatory before presenting it in class. To book a time slot for this meeting, please use the following link by Monday evening as well:

<https://terminplaner6.dfn.de/b/96838838873d1ad7791b2b32d7d8b9a3-1699837>

#### Exercise 1:

A connection game is a congestion game with  $n$  agents and an undirected graph  $G = (V, E)$ . Every agent  $i$  is associated with a subset of vertices  $V_i \subseteq V$ . The set of strategies  $\Sigma_i$  consists of all connected, acyclic subgraphs  $G'_i$  with  $V'_i = V_i$  and  $E'_i \subseteq (E \cap (V_i \times V_i))$ , for every player  $i$ . Every edge  $e$  is assigned a delay function  $d_e(n_e) : \{1, \dots, n\} \rightarrow \mathbb{Z}$ , where  $n_e$  is the number of agents  $i$  selecting a subgraph  $G'_i$  with  $e \in E'_i$ .



- Consider the above instance of a connection game with two players. The vertices in  $V_1$  are indicated in orange, while the vertices in  $V_2$  are marked in blue. Let the initial strategy of player 1 be given by the subgraph  $G'_1$  with edges  $E'_1 = \{\{v_1, v_3\}, \{v_1, v_4\}, \{v_2, v_3\}\}$ . Player 2 chooses subgraph  $G'_2$  with  $E'_2 = \{\{v_1, v_5\}, \{v_2, v_5\}, \{v_4, v_5\}\}$  as his strategy. Perform best-response improvement steps until a pure Nash equilibrium is reached. Player 1 should deviate first.
- Prove that every sequence of best-response improvement steps in a connection game converges in  $O(n^2 \cdot |E| \cdot |V|)$  many steps.

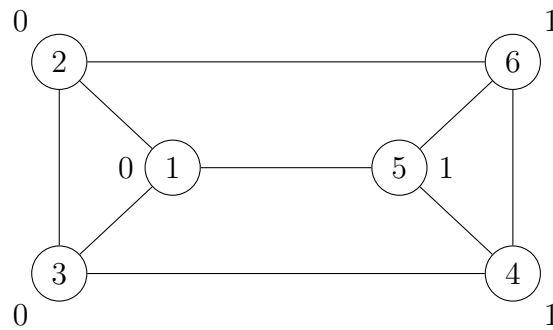
Hint: You can use the following property without proving it.

Let  $G'$  be the strategy of agent  $i$  in state  $S$ , and let  $G''$  be a best response of  $i$  for  $S_{-i}$ . Then, there exists a transforming sequence from  $G'$  to  $G''$ , where in every step, one edge  $e' \in (E' \setminus E'')$  is exchanged by an edge  $e'' \in (E'' \setminus E')$ . For each step, the resulting graph is a feasible strategy for agent  $i$ . In particular, the delay is (weakly) reduced in every step.

**Exercise 2:**

In a *consensus game*, we are given an undirected graph  $G = (V, E)$  with vertex set  $V = \{1, \dots, n\}$ . Each vertex  $i \in V$  is a player and her action consists of choosing a bit  $b_i \in \{0, 1\}$ . Let  $N(i) = \{j \in V \mid \{i, j\} \in E\}$  denote the set of neighbours of player  $i$ , i.e., all players  $j$  connected to  $i$  via an edge. Furthermore, let  $\mathbf{b} = (b_1, \dots, b_n)$  be the vector of players' choices. The loss  $D_i(\mathbf{b})$  for player  $i$  is the number of neighbours that she disagrees with, i.e.,

$$D_i(\mathbf{b}) = \sum_{j \in N(i)} |b_j - b_i|.$$



- a) Calculate the loss  $D_i$  of player 1 for the actions depicted in the graph above.
- b) Show that a consensus game represented as an undirected Graph  $G$  can also be modeled as a congestion game  $\Gamma$ . To this end, specify the tuple  $\Gamma = (N, R, (\Sigma_i)_{i \in N}, (d_r)_{r \in R})$  and show that the loss  $D_i$  coincides with the cost  $c_i$ .
- c) Prove that in a congestion game modeling a consensus game with  $|V| = n$  players all improvement sequences have length  $O(n^2)$ .