

Algorithmic Game Theory

Summer Term 2026

Exercise Set 3

If you would like to submit your solutions for this problem set, please send them via email to aheuser1@uni-bonn.de by Monday evening. Submitting solutions in groups is also possible.

If you would like to present one of your solutions in class, please use the following link to book a presentation slot by Monday evening:

<https://terminplaner6.dfn.de/b/655809818db612a2e76d27a1806653d6-1724946>

A short meeting to discuss your solution is mandatory before presenting it in class. To book a time slot for this meeting, please use the following link by Monday evening as well:

<https://terminplaner6.dfn.de/b/fb796dc5dbc59044ea98f2875e3840a6-1724953>

Exercise 1:

Consider the local search problem *Positive Not-All-Equal kSat* (Pos-NAE- k SAT) which is defined the following way:

Instances: Propositional logic formula with n binary variables x_1, \dots, x_n that is described by m clauses c_1, \dots, c_m . Each clause c_i has a weight $w_i \in \mathbb{N}$ and consists of exactly k literals, which are all positive (i.e., the formula does not contain any negated variable \bar{x}_i).

Feasible solutions: Any variable assignment $s \in \{0, 1\}^n$

Objective function: Sum of weights of clauses c_i in which not all literals are mapped to the same value.

Neighbourhood: Assignments s and s' are *neighbouring* if they differ in the assignment of a single variable.

You can assume that Pos-NAE- k SAT is in PLS. Now:

- (a) Show that $\text{Pos-NAE-2SAT} \leq_{PLS} \text{MaxCut}$
- (b) Show that $\text{Pos-NAE-3SAT} \leq_{PLS} \text{Pos-NAE-2SAT}$

Exercise 2:

We define a Congestion Game to be *symmetric*, if $\Sigma_1 = \dots = \Sigma_n$. Let $PNE_{\text{Cong. Game}}$ and $PNE_{\text{Sym. Cong. Game}}$ be the local search problems in PLS of finding a pure Nash equilibrium of a general or symmetric Congestion Games, respectively.

Show: $PNE_{\text{Cong. Game}} \leq_{PLS} PNE_{\text{Sym. Cong. Game}}$.

Hint: Add an auxiliary resource for each player with a suitable delay function.

Exercise 3:

We want to derive properties of the sets of correlated and coarse correlated equilibria.

- (a) Show that the set of correlated equilibria of a cost-minimization game Γ is convex, i.e. for two correlated equilibria p, p' and $\lambda \in [0, 1]$, also $\lambda p + (1 - \lambda)p'$ is a correlated equilibrium.
- (b) Show that every correlated equilibrium is also a coarse correlated equilibrium.