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Due date: June 29, 2026 at 8:00 p.m.

# Algorithmic Game Theory

Summer Term 2026

Exercise Set 8

*If you would like to submit your solutions for this problem set, please send them via email to aheuser1@uni-bonn.de by Monday evening. Submitting solutions in groups is also possible.*

*If you would like to present one of your solutions in class, please use the following link to book a presentation slot by Monday evening:*
https://terminplaner6.dfn.de/b/f77e46e363fae029a677276f25abdbce-1809991
*A short meeting to discuss your solution is mandatory before presenting it in class. To book a time slot for this meeting, please use the following link by Monday evening as well:*
https://terminplaner6.dfn.de/b/795778e9ad8f1bbb8c0275f060f477cd-1809997

**Exercise 1:**
An *all-pay auction* is a single-item auction defined in almost the same manner as a first-price auction: Each bidder reports a bid $b_i \geq 0$. The bidder with the highest bid wins the item. However, every bidder pays their own bid regardless of whether they win the item or not.

Derive the symmetric Bayes-Nash equilibrium of an all-pay auction with 2 bidders whose valuations are distributed uniformly on the unit interval, i.e. $v_i \sim Unif([0,1])$ for $i = 1, 2$. You may assume that $\beta$ is invertible and differentiable.

**Exercise 2:**
Consider the following single-item auction: Each bidder reports a bid $b_i \geq 0$. The bidder $i$ with the highest bid wins the item and pays *half* their bid $b_i$.

(a) Show that if we only consider two bidders and valuations are drawn uniformly from $[0,1]$, then truthful bidding is a Bayes-Nash equilibrium.

(b) Show that this mechanism is not dominant-strategy incentive compatible.

(c) Show that this mechanism is $(\frac{1}{2}, 1)$-smooth.

**Exercise 3:**
Consider $m$ items and $n$ bidders. We define a generalization of Walrasian equilibria: Let $S = (S_1, \ldots, S_n)$ be an allocation of items to bidders and $q \in \mathbb{R}^m_{\geq 0}$ a price vector. We call the pair $(q, S)$ an $\epsilon$-*approximate Walrasian equilibrium* if unallocated items have price 0, every bidder $i$ has non-negative utility $v_i(S_i) - \sum_{j \in S_i} q_j \geq 0$, and every bidder receives items within $\epsilon$ of their favorite bundle, i.e., $v_i(S_i) - \sum_{j \in S_i} q_j \geq v_i(S'_i) - \sum_{j \in S'_i} q_j - \epsilon$ for every bundle $S'_i$.

Prove an approximate version of the First Welfare Theorem: If $(q, S)$ is an $\epsilon$-approximate Walrasian equilibrium, then the social welfare of an optimal allocation $S^*$ cannot surpass the one of $S$ by more than $\min\{m, n\} \cdot \epsilon$.