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Due date: July 6, 2026 at 8:00 p.m.

Algorithmic Game Theory

Summer Term 2026

Exercise Set 9

If you would like to submit your solutions for this problem set, please send them via email to aheuser1@uni-bonn.de by Monday evening. Submitting solutions in groups is also possible.

If you would like to present one of your solutions in class, please use the following link to book a presentation slot by Monday evening:
https://terminplaner6.dfn.de/b/31875937bcf6c4c18c4fd277aa24e535-1820879
A short meeting to discuss your solution is mandatory before presenting it in class. To book a time slot for this meeting, please use the following link by Monday evening as well:
https://terminplaner6.dfn.de/b/c0813d676f8421555108e5ca52c7d949-1820895

Exercise 1:
Consider the setting from lecture 18, where we want to sell a single item under limited information (for $v_i \sim \mathcal{D}_i$, we know the distributions $\mathcal{D}_i$ but not the realizations v_i).
We offer the item for a price p. The customers arrive sequentially and buy the item if $v_i \geq p$. Show that if we set p so that $\mathbf{Pr}\left[\text{do not sell item}\right] = \frac{1}{2}$, we attain at least half of the optimal welfare.

Exercise 2:
Determine the virtual value function φ of the following probability distributions.

(a) Uniform distribution on the interval $[a, b]$.

(b) Exponential distribution with rate $\lambda > 0$ (defined on $[0, \infty)$).

(c) The distribution given by the cumulative distribution function $F(v) = 1 - \frac{1}{(v+1)^c}$ defined on the interval $[0, \infty)$, where $c > 0$ is considered to be an arbitrary constant.

Which of the stated distributions are regular?

Exercise 3:
Once again, consider a single-item auction with two bidders whose valuations are drawn independently from a uniform distribution over $[0, 1]$.

(a) Prove that the random variables $\varphi_i(v_i)$ are distributed according to a uniform distribution on $[-1, 1]$.

(b) Define a second-price auction with *reserve price* p. Let v_1 and v_2 be the valuations of the bidders. The allocation and payment rule will be determined according to the following cases:

1. $\min\{v_1, v_2\} \geq p$: Like in the second price auction.
2. $\max\{v_1, v_2\} < p$: Nobody gets the item and no payments.
3. $v_1 \geq p > v_2$: Bidder 1 gets the item and has to pay p.
4. $v_2 \geq p > v_1$: Analogous to 3.

Utilize subtask (a) and the results of the lecture in order to determine the expected revenue of a second-price auction with reserve price $p \in [0, 1]$.

Exercise 4:
We want to discuss non-truthful mechanisms. Therefore, consider a single-item first-price auction with n bidders whose values are drawn uniformly at random from $[0, 1]$.

(a) Show that each bidder reporting a $\frac{n-1}{n}$-fraction of their actual value is a Bayes-Nash equilibrium.

(b) Compute the expected revenue of the first-price auction at equilibrium.