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November 27, 2018
Due: lecture on December 11, 2018

Algorithms and Uncertainty

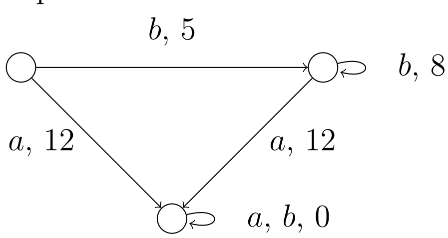
Winter Semester 2018/19

Exercise Set 6

There are no tutorials on December 5 and 6.

Exercise 1: (4 Points)
We generalize the cardinality-robust version of Vertex Cover as follows. Partition the set of possible edges in the graph into sets $B_1, \ldots, B_\ell$. The scenario set $\mathcal{E}$ contains all sets E for which $|E \cap B_i| = k_i$. Extend the approximation algorithm from the lecture and outline its analysis.

Exercise 2: (1+1+1 Points)
We consider a Markov decision process with $\mathcal{S} = \{1, 2, 3\}$, $\mathcal{A} = \{a, b\}$. The state transitions are deterministic as displayed in this diagram; the numbers in the edge labels are the respective rewards.



We consider an infinite time horizon with discount factor $\gamma = \frac{1}{2}$.

(a) Give an optimal policy and the function $s \mapsto V^*(s)$.

(b) Perform the first six steps of value iteration starting from $W^{(0)} = (0, 0, 0)$.

(c) Perform policy iteration until convergence starting from the policy that always uses action a.

Exercise 3: (4 Points)
We define a more cautious version of value iteration. It uses the operator T', which is defined by $T'(W) = \eta T(W) + (1 - \eta) W$ for an arbitrary $\eta \in (0, 1)$. Show that this algorithm also converges to the unique fixed point of T.

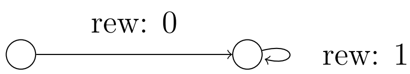
Exercise 4 on the next page.

Exercise 4: (3+2+2+2 Points)

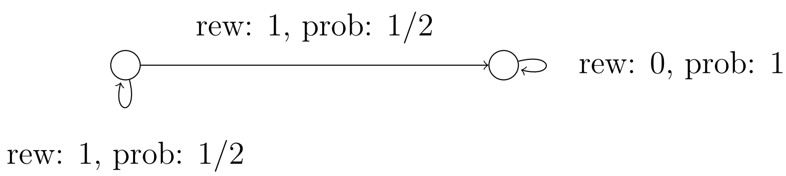
For the following single-armed bandits, give the fair charges of all states. Unless states otherwise, the transitions are deterministic. Justify your statements if necessary. For part (a), consider $\gamma = \frac{1}{2}$; for the remaining parts an arbitrary $\gamma \in (0, 1)$.

(a)

(b)



(c)



(d)