MA-INF 1203 Discrete and Computational Geometry

Wintersemester 2019/20

Assignment 3

Deadline: **29** October before noon (To be discussed: **29/30**. October 2019)

1 Centerpoint

Prove that for any k finite sets $A_1, \dots, A_k \subset \mathbb{R}^d$, where $1 \leq k \leq d$, there exists a $(k-1)$-flat such that every hyperplane containing it has at least $\frac{|A_i|}{d+1}$ points of A_i in each of its two closed half-spaces for all $i = 1, 2, \dots, k$.

2 Enclosing disk

Each set $X \subset \mathbb{R}^2$ of diameter at most 1 (any two points have distance at most 1) is contained in some disk of radius $1/\sqrt{3}$.

a) Prove the statement for $|X| = 3$.

b) Prove the statement for all finite sets $X \subset \mathbb{R}^2$ using Helly's Theorem.

3 Duality and line segments

a) Consider $n \geq 3$ line segments in the plane, such that none of them contains 0 but they all lie on lines passing through 0. Show that if every 3 among such segments can be intersected by a single line, then all the segments can be simultaneously intersected by a line. Use the projective duality transform.

b) Let $n \geq 3$, be vertical segments in the plane. Show that if every 3 among such segments can be intersected by a single line, then all the segments can be simultaneously intersected by a line. Use the other duality transform.