Thomas Kesselheim
Alexander Braun

January 28, 2021

Algorithmic Game Theory

Winter Term 2020/21

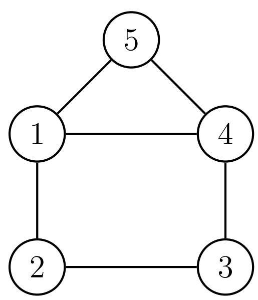
Tutorial Session - Week 11

Exercise 1:
Consider the following instance of the house-allocation problem. There are six agents $a, \dots, f$ and their preferences are given by:

$$a : b > d > f > e > c > a\,, \qquad b : d > a > c > e > f > b\,,$$
$$c : e > f > a > c > b > d\,, \qquad d : e > a > b > c > d > f\,,$$
$$e : f > e > c > b > d > a\,, \qquad f : d > a > b > c > f > e\,.$$

Find a stable allocation π using the Top Trading Cycle Algorithm.

Exercise 2:
Consider the problem of Pairwise Kidney Exchange by Matching from Lecture 21. The graph below depicts an instance of agents (that is, patient-donor pairs) and possible pairwise exchanges, i,e. nodes represent patient-donor pairs with an edge connecting two nodes if an exchange between the two patient-donor pairs is possible.



(a) Use the mechanism of Section 4 from Lecture 21 and consider agents in ascending order of agent indices (which is independent of the reports) to determine the set of maximum matchings M_5.

(b) Now, let us show that the given matching algorithm is not DSIC if the order in which the algorithm processes the agents depends on their reports. For this purpose, consider a modified algorithm that processes agents in ascending order of node degree (tie-breaking in favor of the agent with the smallest index) and verify that agent 4 can do better by misreporting in the given instance.