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November 6, 2023
Due: November 13, 2023 at 10:00 p.m.

Algorithms and Uncertainty

Winter Term 2023/24

Exercise Set 5

If you want to hand in your solutions for this problem set, please send them via email to anna.heuser@uni-bonn.de *by Monday evening – make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to anna.heuser@uni-bonn.de *containing the* ***task*** *which you would like to present and in* ***which of the tutorials*** *you would like to do so. Deadline for the email is Monday, 10:00 pm. Please note that the tasks will be allocated via a first-come-first-served procedure, so sending this email earlier than Monday evening is highly recommended.*

Exercise 1: (4 Points)
Let us consider a generalization of the version of Markov decision processes covered in the lecture. For every state $s \in \mathcal{S}$, only a subset of the actions $\mathcal{A}_s \subseteq \mathcal{A}$, $\mathcal{A}_s \neq \emptyset$, is available. Devise an algorithm that computes an optimal policy for a finite time horizon T, show its correctness, and give a bound on its running time.

Exercise 2: (2 Points)
Consider the cost-minimization variant of the optimal stopping problem in which we know the prior distributions. In step i, we can stop the sequence at cost c_i. We have to stop the sequence at some point and want to minimize the cost for doing so.
Show that there is **no** $\alpha < \infty$ such that for all distributions the optimal policy has cost at most $\alpha \mathbf{E}[\min_i c_i]$.

Hint: It sufficies to consider $n = 2$.

Exercise 3: (5 Points)
We consider the following stochastic decision problem: There are n boxes; box i contains a prize of 1 Euro with probability q_i and is empty otherwise. The game ends when we have found a non-empty box. That is, the final prize is either 0 Euros or 1 Euro. At each point in time, we can also decide to stop playing. We can open as many boxes as we like but opening box i costs c_i Euros. Find an optimal policy.

Hint: It can be useful to consider the cases $n = 1$ and $n = 2$ first.