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December 4, 2023
Due: December 11, 2023 at 10:00 p.m.

Algorithms and Uncertainty

Winter Term 2023/24

Exercise Set 9

If you want to hand in your solutions for this problem set, please send them via email to anna.heuser@uni-bonn.de *by Monday evening – make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to anna.heuser@uni-bonn.de *containing the* ***task*** *which you would like to present and in* ***which of the tutorials*** *you would like to do so. Deadline for the email is Monday, 10:00 pm. Please note that the tasks will be allocated via a first-come-first-served procedure, so sending this email earlier than Monday evening is highly recommended.*

Exercise 1: (4 Points)

Consider a policy π' that would be optimal if fair caps were $\sigma'_1, \dots \sigma'_n$. Show that, if $|\sigma_i - \sigma'_i| \leq \gamma$ for all boxes i, then the expected reward of π' with respect to actual fair caps $\sigma_1, \dots, \sigma_n$ is at least $V^* - n\gamma$.

Exercise 2: (4 Points)

Given a distribution $\mathcal{D}$ of values in $[0,1]$ and cost $c \in [0,1]$, let $\sigma_{\mathcal{D}}$ denote the corresponding fair cap. Additionaly let $\tilde{\sigma}_{\mathcal{D}}$ denote the fair cap with respect to the empirical distribution defined by T samples drawn from $\mathcal{D}$. Show that for any T and any $\epsilon > 0$ there exists a distribution $\mathcal{D}_{(T,\epsilon)}$ and cost $c_{(T,\epsilon)}$ such that $\left|\sigma_{\mathcal{D}_{(T,\epsilon)}} - \tilde{\sigma}_{\mathcal{D}_{(T,\epsilon)}}\right| \geq 1 - \epsilon$ with probability at least $1 - \frac{1}{T}$.

Exercise 3: (3+4 Points)

a) We have a bucket of 100 balls. Some of them are green, the others are red. We want to find out if the bucket contains more red or green balls. For this, we draw n balls uniformly at random and with replacement from the bucket. Use Hoeffding's inequality to show that the probability for us to guess correctly is at least $1 - \exp\left(-\frac{8}{25}n\right)$, if the bucket contains 10 red and 90 green balls.

b) Now we have a second bucket of 100 balls. We want to find out, which bucket has more green balls and thus draw n samples from the second bucket as well. Use Hoeffding's inequality and the union bound to show that the probability for us to guess correctly is at least $1 - 2\exp\left(-\frac{2}{25}n\right)$, if the first bucket contains 90 green balls and the second bucket contains 50 green balls.