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January 22, 2024
Due: January 29, 2024 at 10:00 p.m.

Algorithms and Uncertainty

Winter Term 2023/24

Exercise Set 14

If you want to hand in your solutions for this problem set, please send them via email to anna.heuser@uni-bonn.de *by Monday evening – make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to anna.heuser@uni-bonn.de *containing the* ***task*** *which you would like to present and in* ***which of the tutorials*** *you would like to do so. Deadline for the email is Monday, 10:00 pm. Please note that the tasks will be allocated via a first-come-first-served procedure, so sending this email earlier than Monday evening is highly recommended.*

Exercise 1: (2+5+3 Points)

We want to consider the following Hypergraph-variant of Secretary Matching. Again we have n applicants arriving online and we have multiple positions that we hire for. However now every position consists of two jobs and we can combine any two jobs to form a position.
In the graph version of the problem this corresponds to a graph with every edge covering exactly 2 offline vertices.
Show that the algorithm from lecture 26 is $\frac{1}{4}$-competitive for this variant with random arrival order, a good choice of τ and $n \to \infty$.

(a) Show that the the expected weight of the tentatively matched edge in any step is still at least $\frac{1}{n} w(\mathrm{OPT}(G))$.

(b) Show that the conditional probability that the tentatively matched edge in step t can actually be added to the matching is at least $\frac{(\tau-1)\tau}{(t-2)(t-1)}$.

(c) Use (a) and (b) as well as $\sum_{t=\tau+1}^{n} \frac{1}{(t-2)(t-1)} \geq \int_{\tau+1}^{n} \frac{1}{x^2} \mathrm{d}x$ to finish the proof.

Exercise 2: (4 Points)

Let $G = (V, E)$ be a graph with edge capacities $(c_e)_{e \in E}$, a source $s \in V$ and a sink $t \in V$. Let $\mathcal{P}$ be the set of all s-t paths in G and $|\mathcal{P}| \leq m = |E|$. Show that, if $T \geq \frac{4}{\epsilon^2} |\mathcal{P}| \log m$, the algorithm from lecture 24 then guarantees $\sum_{P \in \mathcal{P}} x_P \geq (1 - \epsilon) F^*$ when using Multiplicative Weights as the experts algorithm with $\eta = \frac{\epsilon}{2}$.