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Algorithms and Uncertainty

Winter Term 2023/24

Tutorial Session - Week 1

If you do not know each other yet, each of you could start with a very quick introduction: What's your name? Do you study Computer Science or maybe something else (Maths, Economics,...)? Do you have any prior knowledge in Algorithms and Uncertainty?
Afterwards, you are supposed to discuss the exercises on this sheet. Note that you should see this also as a chance to talk about definitions, proof ideas and techniques used in the lecture in addition to only working out a formal solution for the tasks. If you do not know a definition or theorem by hard, feel free to open the lecture notes and have a look. Further, if you have any questions, I will drop by to discuss possible issues with you.

If there is some remaining time at the end of the tutorial, you can share your ideas on the tasks with the whole group.

Exercise 1:
We want to recall the basics of linear programming. Therefore, we consider the *Vertex Cover* problem: The task is to cover edges in a graph where an edge can be covered by its incident vertices. More formally, a vertex cover is a set of vertices $S \subseteq V$ such that for all $e = \{u, v\} \in E$ either $u \in S$ or $v \in S$. We are interested in finding a vertex cover of minimum size. For now, we restrict to bipartite graphs, i.e. graphs $G = (V, E)$ with $V = L \cup R$.

(a) Give the integer program of the Vertex Cover problem and its LP relaxation.

(b) Give the dual program to the LP from (a).

Exercise 2:
We consider the online maximum bipartite matching problem: Assume there is a bipartite graph $G = (L \cup R, E)$, but initially, we only know the set L. In each round, one vertex in R is revealed with all its incident edges. We now have to decide immediately and irrevocably if we want to select one of these edges or leave r unmatched. As in the offline matching problem, any vertex in L can only be matched to at most one vertex in R.

A very simple algorithm is the following one: Fix an ordering of vertices in L. Whenever a vertex $r \in R$ is revealed, if there is still an unmatched neighbor, match it to the unmatched neighbor of smallest index.

Show that $|\text{ALG}| \geq \frac{1}{2}|\text{OPT}|$ for any input graph G and any arrival order of vertices in R, where $|\text{ALG}|$ denotes the size of the matching computed by the algorithm and $|\text{OPT}|$ is the maximum offline matching in G.