

Algorithms and Uncertainty

Winter Term 2023/24

Tutorial Session - Week 10

Exercise 1:

Recall the regret definition from the lecture: $\text{Regret}^{(T)} = L_{\text{Alg}}^{(T)} - \min_i \sum_{t=1}^T \ell_i^{(t)}$.

We want to understand the order of minimum and sum in the second term. Therefore, work on the following tasks.

- (a) Use Yao's principle to show that for every (randomized) algorithm there is a sequence $\ell^{(1)}, \dots, \ell^{(T)}$ such that $L_{\text{Alg}}^{(T)} \geq (1 - \frac{1}{n}) T$ but $\sum_{t=1}^T \min_i \ell_i^{(t)} = 0$.
- (b) Discuss the importance of the order of sum and minimum in the regret definition using your results from (a).

Exercise 2:

Show that every no-regret algorithm in the experts setting has to be randomized. Consider the case $n = 2$ and for every deterministic algorithm construct a sequence such that $L_{\text{Alg}}^{(T)} = T$ and $\min_i L_i^{(T)} \leq \frac{T}{2}$.