

Algorithms and Uncertainty

Winter Term 2025/26

Tutorial Session - Week 8

Exercise 1:

We want to consider the Stochastic Vertex Cover problem. The edge set $A \subseteq E$ is uncertain, but drawn from a known probability distribution. The probability that the edge set is $A \subseteq E$ is given by p_A . Our goal is to compute a Vertex Cover of minimum cost for the graph $G = (V, A)$. Before A is revealed, we have to pay c_v^I for v , afterwards $c_v^{II} \geq c_v^I$.

- (a) Derive an LP such that every policy corresponds to a feasible solution. Consider variables x_v denoting if v is picked in the first stage and $y_{A,v}$ if the edge set is A and v is picked in the second stage.
- (b) In order to compute a feasible policy, we use the following algorithm which uses an optimal LP solution (x^*, y^*) of the LP above:
 - 1. In the first stage, pick all vertices for which $x_v^* \geq \frac{1}{4}$.
 - 2. In the second stage, when knowing A , pick all vertices for which $y_{A,v}^* \geq \frac{1}{4}$.

Show that this algorithm always computes a feasible policy.

- (c) Show that the expected cost of the computed policy is at most 4-times the expected cost of the optimal policy.