Discrete and Computational Geometry, SS 14
Exercise Sheet "5": Randomized Algorithms for Geometric Structures III and Abstract Voronoi Diagram
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until* **Tuesday 20th of May, 14:00 pm**. *There will be a letterbox in the LBH building.*
- *You may work in groups of at most two participants.*
- *Please contact Hilko Delonge, hilko.delonge@uni-bonn.de, if you want to participate and have not yet signed up for one of the exercise groups.*
- *If you are not yet subscribed to the mailing list, please do so at https://lists.iai.uni-bonn.de/mailman/listinfo.cgi/lc-dcgeom*

Exercise 13: 3D Convex Hull by Conflict Lists (4 Points)

Given a set N of n half-spaces each of which is defined by a hyperplane in the 3D space, a 3D convex hull $H(N)$ of N is the common intersection of all half-spaces of N, Let $S_1, S_2, \ldots, S_n$ be a random sequence of N, and let N^i be $\{S_1, S_2, \ldots, S_i\}$. Please develop a randomized algorithm to construct $H(N)$ by computing $H(N^4), H(N^5), \ldots, H(N^n)$ iteratively using the conflict lists. In other words, for $i \geq 4$, obtain $H(N^{i+1})$ from $H(N^i)$ by adding S_{i+1}. To simplify the description, we assume there is no four half-spaces whose defining hyperplanes intersect at the same point.

1. Define a configuration in $H(N^i)$
2. Define a conflict relation between a configuration in $H(N^i)$ and a half-space in $N = \backslash N^i$
3. Describe the insertion of S^{i+1} using the conflict lists
4. Describe the updation of the conflict lists

5. Prove the complexity of this randomized incremental construction

(Hint: Let cap(S^{i+1}) be the intersection between edges of $H(N^i)$ and the complement of S^{i+1}. For an edge e of $H(N^{i+1})$ which does not belong to $H(N^i)$, e and cap(S^{i+1}) form a cycle, and if a half-space $I \in S \setminus N^{i+1}$ intersects e, I must intersect one of edges of the cycle except e.)

Exercise 14: 3D Convex Hull by History Graph (4 Points)

Given a set N of n half-spaces each of which is defined by a hyperplane in the 3D space, a 3D convex hull $H(N)$ of N is the common intersection of all half-spaces of N, Let $S_1, S_2, \ldots, S_n$ be a random sequence of N, and let N^i be $\{S_1, S_2, \ldots, S_i\}$. Please develop a randomized algorithm to construct $H(N)$ by computing $H(N^4), H(N^5), \ldots, H(N^n)$ iteratively using the history graph. In other words, for $i \geq 4$, obtain $H(N^{i+1})$ from $H(N^i)$ by adding S_{i+1}. To simplify the description, we assume there is no four half-spaces whose defining hyperplanes intersect at the same point.

1. Define a configuration in $H(N^i)$
2. Define the parent and child relation between a configuration in $H(N^i) \setminus H(N^{i+1})$ and a configuration in $H(N^{i+1}) \setminus H(N^i)$
3. Describe the insertion of S^{i+1} using the history graph
4. Prove the complexity of this randomized incremental construction

(Hint:

- Let cap(S^{i+1}) be the intersection between edges of $H(N^i)$ and the complement of S^{i+1}. For an edge e of $H(N^{i+1})$ which does not belong to $H(N^i)$, e and cap(S^{i+1}) form a cycle, and if a half-space $I \in S \setminus N^{i+1}$ intersects e, I must intersect one of edges of the cycle except e.
- There are three kind of edges in $H(N^{i+1})$, and the last two belong to $H(N^{i+1}) \setminus H(N^i)$
  1. an edge is also an edge of $H(N^i)$
  2. an edge is a part of an edge of $H(N^i)$

3. an edge is completely new and contained in the hyperplane defining S^{i+1}

- There are two kind of edges in $H(N^i) \setminus H(N^{i+1})$
  1. an edge is fully contained in S^{i+1}.
  2. an edge is only partially contained in S^{i+1}, and the intersection between it and the complement of S^{i+1} is an edge of $H(N^i)$ (the second kind edge of $H(N^{i+1})$).

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Exercise 15: Line segments and Abstract Voronoi diagram (4 Points)

Consider a set S of n disjoint line segments, and let $\mathcal{J}$ be the $\binom{n}{2}$ bisecting curves among S. Please prove the bisecting system $(S, \mathcal{J})$ is admissible, i.e., the corresponding Voronoi diagram is an abstract Voronoi diagram.