

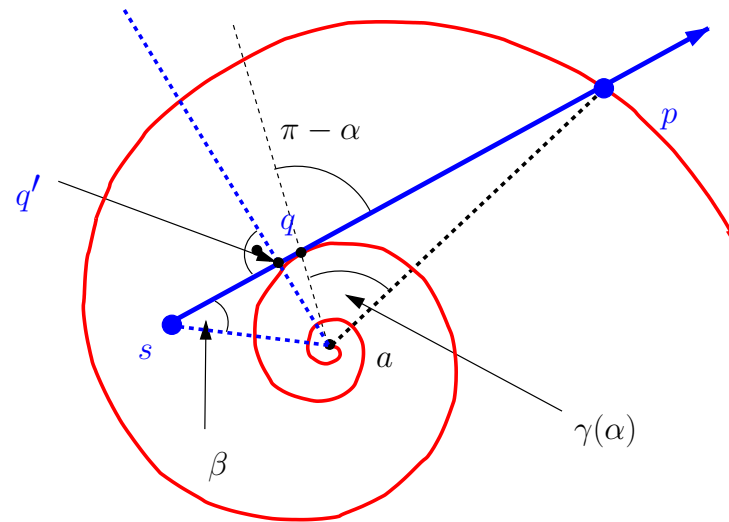
Online Motion Planning MA-INF 1314

General rays! Searching in streets!

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WH: General rays, spiral search

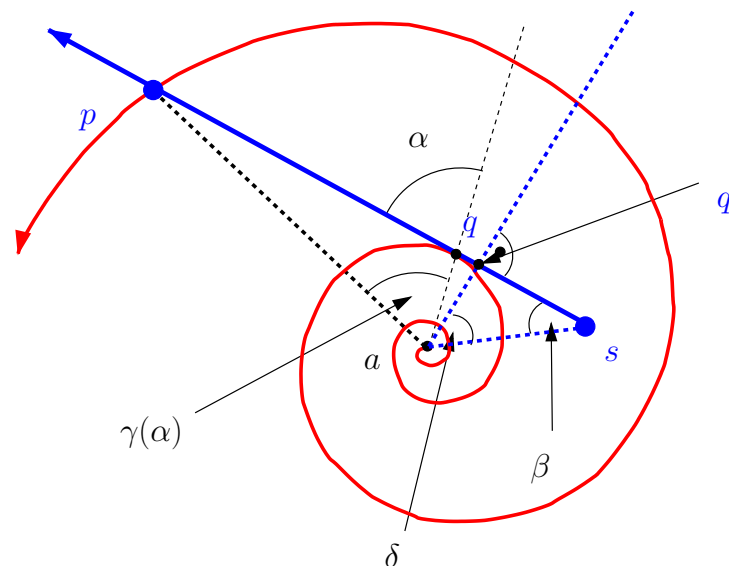
- **Lemma** Worst-case: Ray is a tangent ■
- **Lemma** Ratio $C(\beta, \alpha) = C(\alpha) \sin(\beta) + \cos(\beta)$ ■
- Ratio depends on α (angle $\gamma(\alpha)$) ■



WH: Minimize worst-case for $s = q'$

- **Lemma Solve Equation:** $\frac{\sin \alpha}{\sin(\alpha - \gamma(\alpha))} = E^{\cot \alpha(2\pi + \gamma(\alpha))}$ ■
- **Optimize:**

$$C(\alpha) = \frac{|\Pi_a^p| + |pq'|}{|aq'|} := \left(\frac{1}{\sin \alpha \cdot \cos \alpha} + \frac{\sin \gamma(\alpha)}{\sin^2 \alpha} \right) E^{b(2\pi + \gamma(\alpha))} + \cot \alpha$$
 ■
- **Maximize:** $g(\beta) := C(\alpha_{\min}) \sin \beta + \cos \beta$! ■



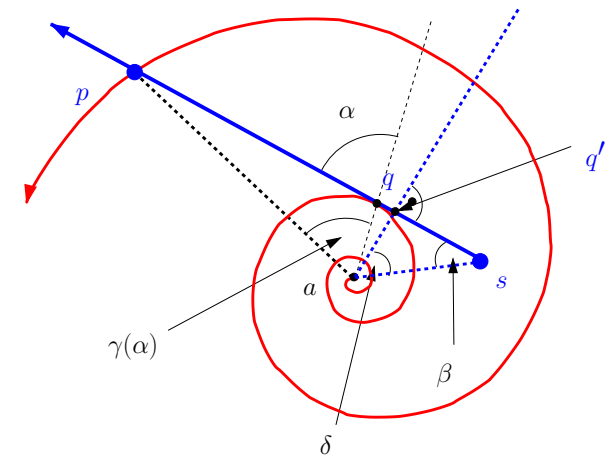
Optimizing the spiral: Theorem

- Ratio: $C(\alpha)$ for $s = q'$ minimal for $\alpha = 1.4575 \dots$ ■
- $C(\alpha) = 22.4908 \dots$ Theo. ■
- Adversary choose β for max.
 $D(\beta, \alpha) = C(\alpha) \sin(\beta) + \cos(\beta)$ ■
- For $\alpha = 1.4575 \dots$

choose $\beta = 1.526363 \dots$

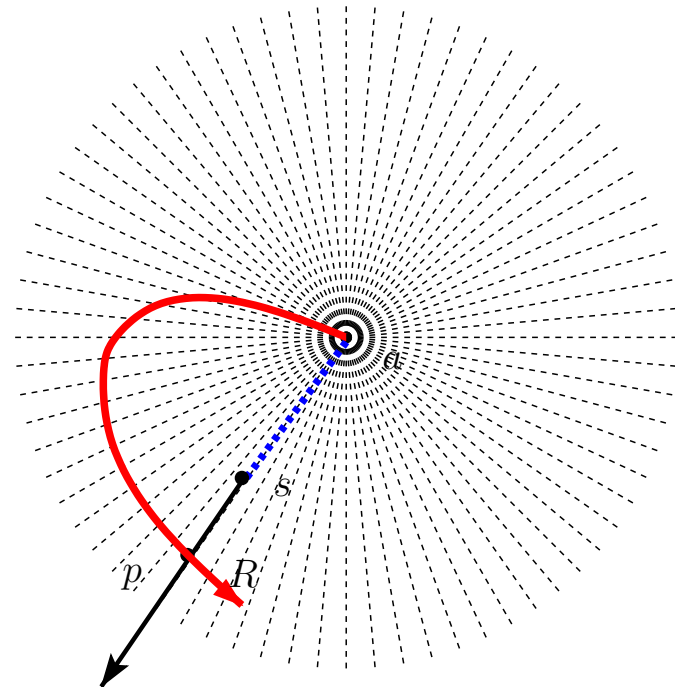
■ $D(\beta, C(\alpha)) = 22.51306056 \dots$

Kor.■



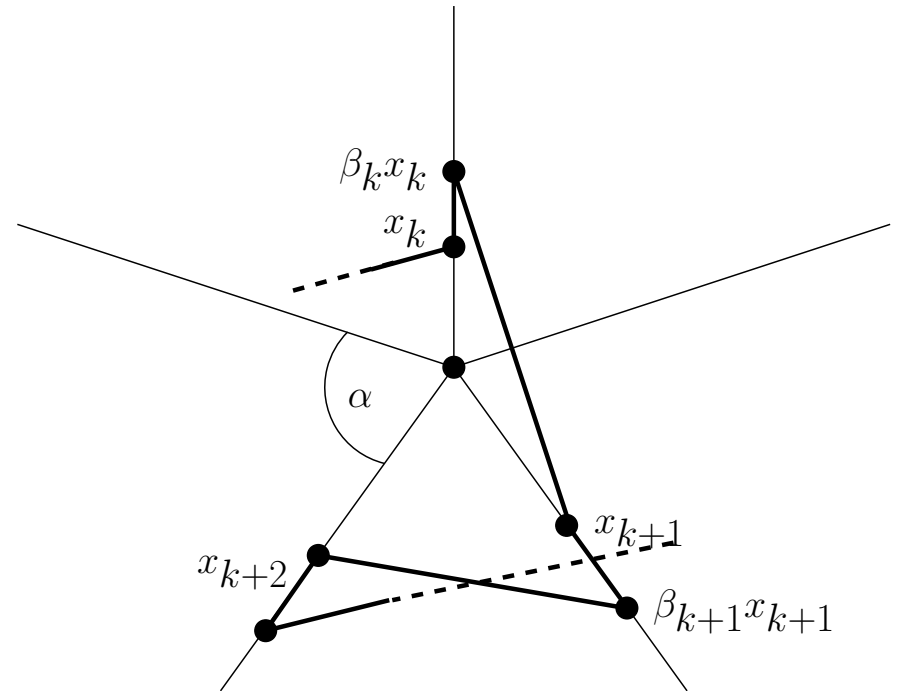
Lower bound: Special example

- General problem of lower bounds
- Special example: Searching for
- a special ray in the plane
- Cross R and detect s ■
- Special case: No move to s ■
- Alpern/Gal (Spiral search):
Ratio $C = 17.289\dots$ ■
- Best ratio among monotone and
periodic strategies■
- "Complicated task": There is
an optimal periodic/monotone
strategy■



Lower bound construction

- n known rays emanating from a ■
- Angle $\alpha = \frac{2\pi}{n}$ ■
- Find s on one of the rays ■
- Also non-periodic and non-monotone strategies ■
- Strategy S : Visits the rays in some order ■
- Hit x_k , leave $\beta_k x_k$ ($\beta_k \geq 1$) ■

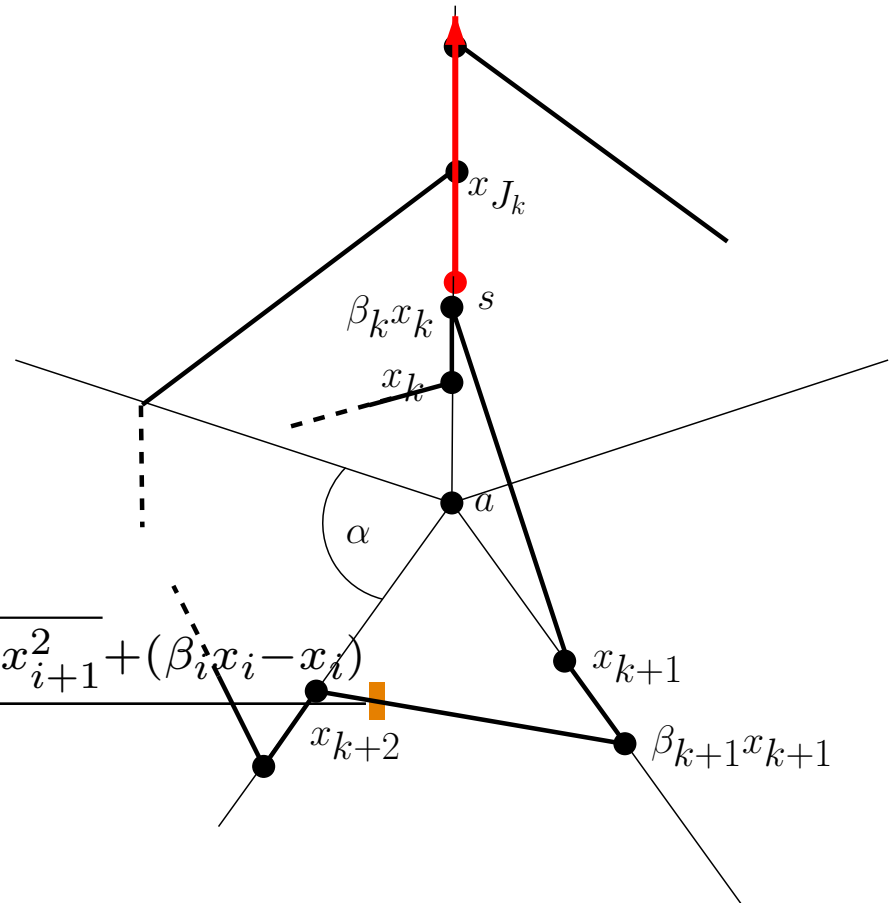


Lower bound construction

- Find s on a ray visited up to $\beta_k x_k$ at the last time, now at x_{J_k}
- Note: Any order is possible
- Worst-case, s close to $\beta_k x_k$
- Ratio: $C(S)$

$$\frac{\sum_{i=1}^{J_k-1} \sqrt{(\beta_i x_i)^2 - 2\beta_i x_i x_{i+1} \cos \gamma_{i,i+1} + x_{i+1}^2} + (\beta_i x_i - x_i)}{\beta_k x_k}$$

- Monotone/Periodic, Funktional??



- Ratio: $C(S)$ **Lower bound construction**

$$\frac{\sum_{i=1}^{J_k-1} \sqrt{(\beta_i x_i)^2 - 2\beta_i x_i x_{i+1} \cos \gamma_{i,i+1} + x_{i+1}^2} + (\beta_i x_i - x_i) J_k x_{J_k}}{\beta_k x_k}$$

- Shortest distance to next ray:

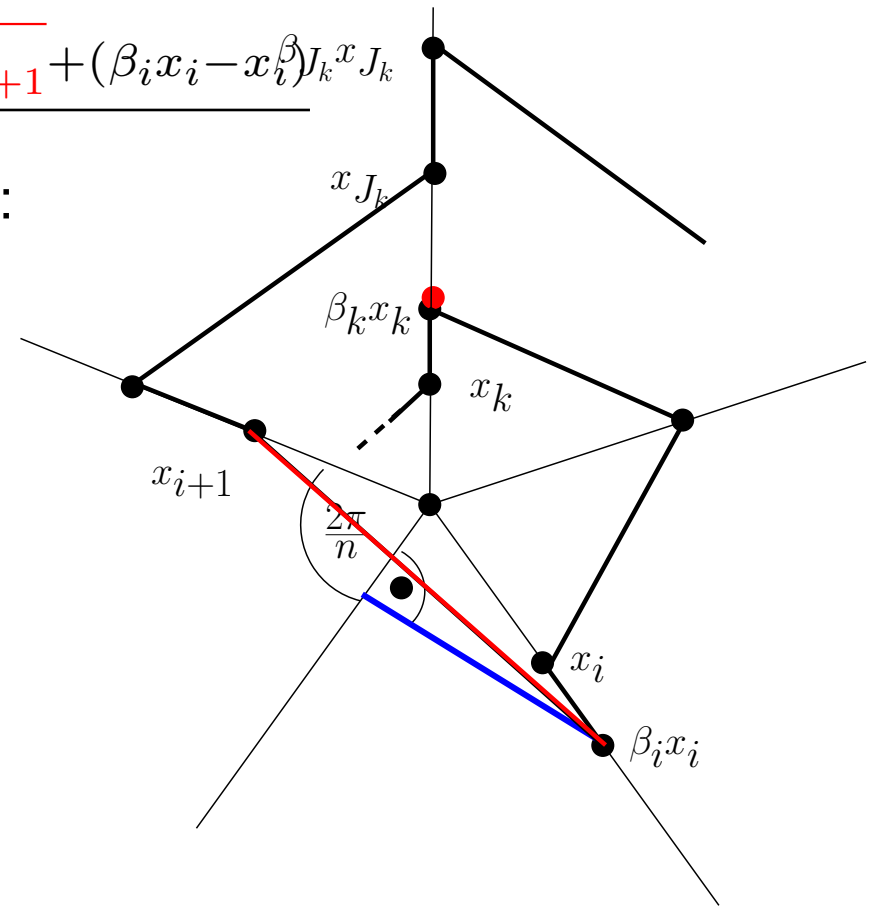
$$\beta_i x_i \sin \frac{2\pi}{n}$$

- Lower bound for

$$\sqrt{(\beta_i x_i)^2 - 2\beta_i x_i x_{i+1} \cos \gamma_{i,i+1} + x_{i+1}^2}$$

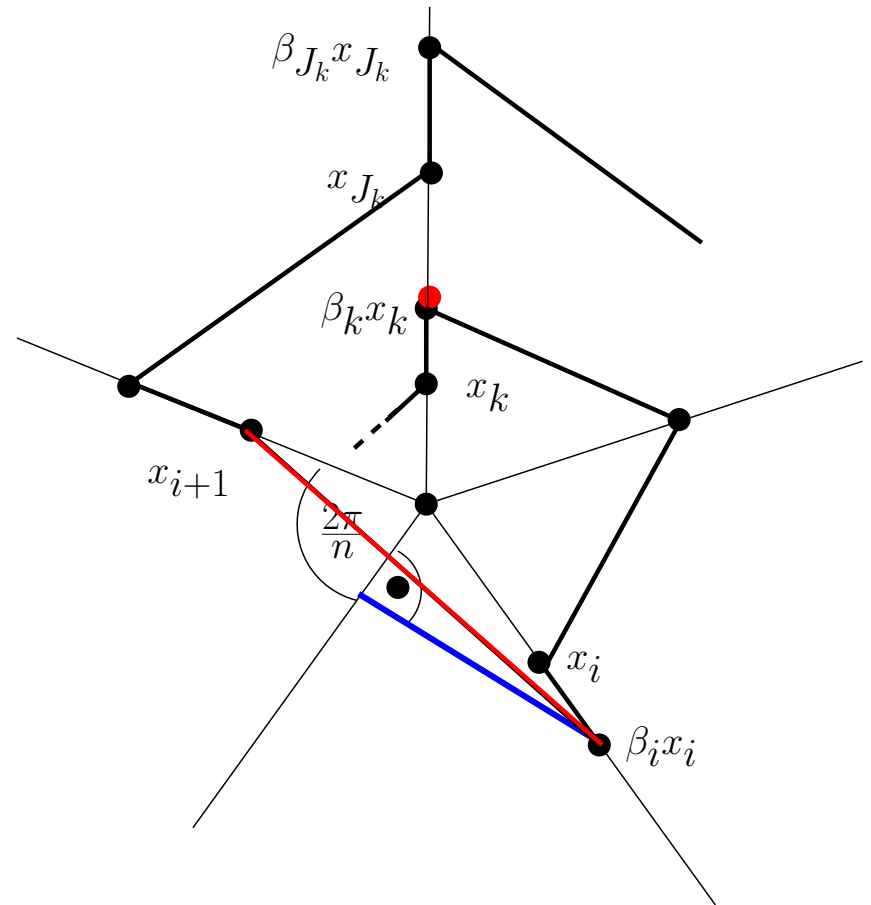
- Lower bound: $C(S) \geq$

$$\sin \frac{2\pi}{n} \frac{\sum_{i=1}^{J_k-1} \beta_i x_i}{\beta_k x_k}$$



Lower bound construction

- Lower bound $\frac{\sum_{i=1}^{J_k-1} f_i}{f_k}$ ■
- Equals functional of standard m-ray search ■
- Optimal strategy: monotone/periodic (Alpern/Gal) ■
- $f_i = \left(\frac{n}{n-1}\right)^i$
- ratio: $(n-1) \left(\frac{n}{n-1}\right)^n$ ■
- $C(S) \geq \sin \frac{2\pi}{n} (n-1) \left(\frac{n}{n-1}\right)^n$ ■

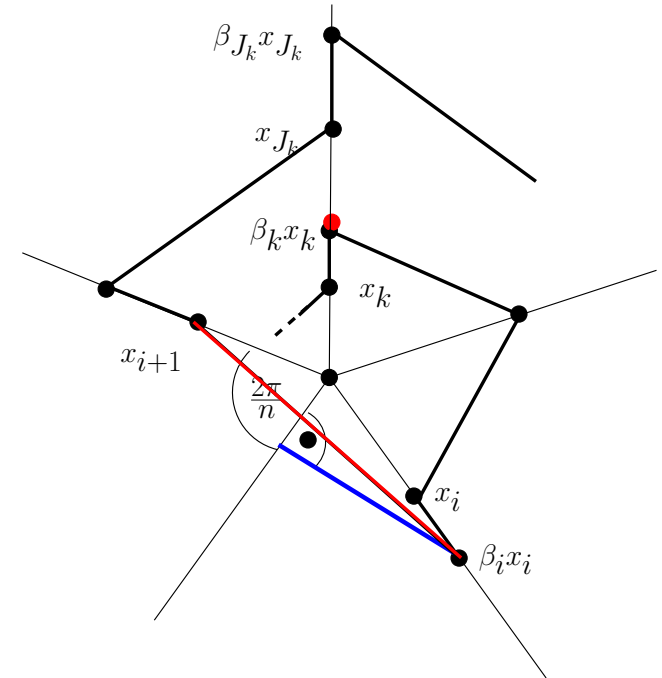


Lower bound construction

- $C(S) \geq \sin \frac{2\pi}{n} (n-1) \left(\frac{n}{n-1}\right)^n$ ■

- $$\lim_{n \rightarrow \infty} (n-1) \left(\frac{n}{n-1}\right)^n \sin \frac{2\pi}{n} =$$

- $$2\pi e = 17.079\dots$$



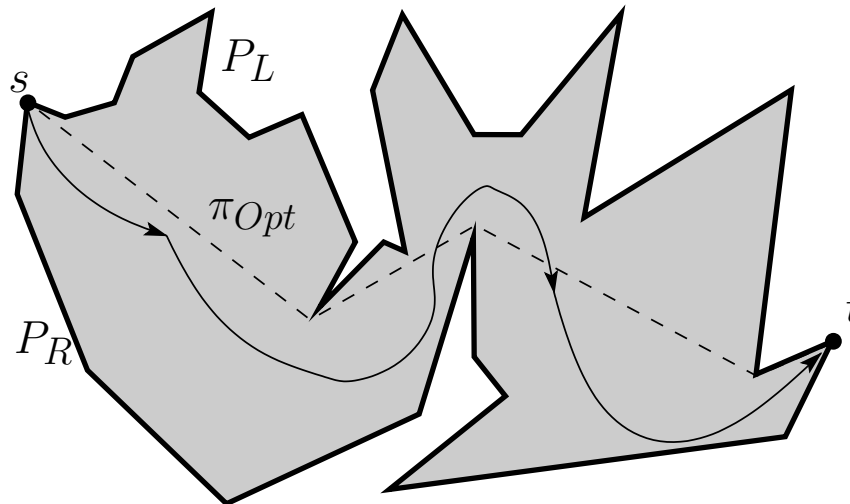
- Lower bound: **Theorem** ■

Summary

- The Window-Shopper-Problem
- Optimal strategy $C = 1.059 \dots$: **Theorem**
- Interesting design technique
- Rays in general
- Lower $C \geq 2\pi e = 17.079 \dots$ (**Theorem**) and upper bound $C = 22.51 \dots$ (**Theorem**)
- Lower bound construction
- Also a lower bound for special case with $C = 17.289 \dots$

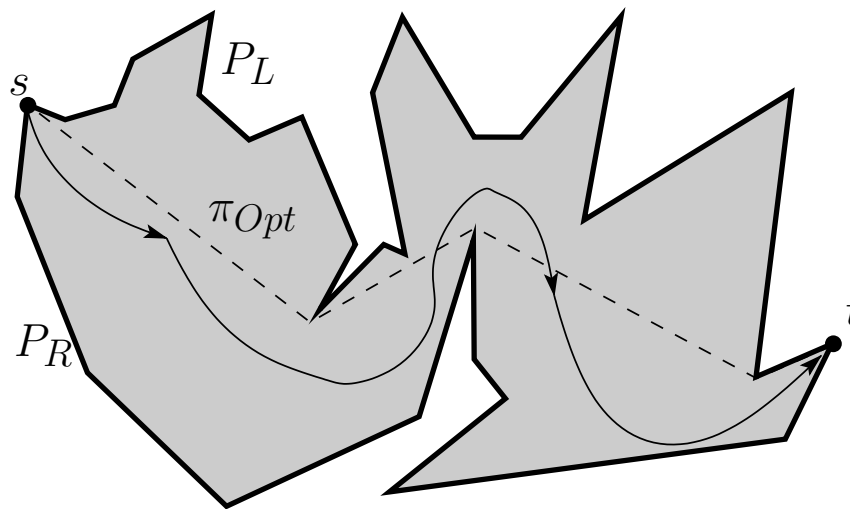
Searching in street polygons

- Searching in simple polygons, visibility
- Subclass: Streets
- Start- and target
- Target t unknown, search for $t!$
- Compare to shortest path to $t!$ ■ Comp. factor!!



Formal definition

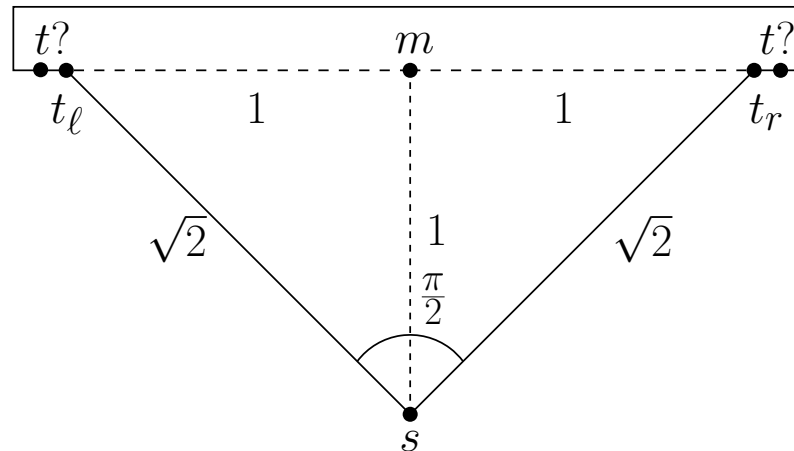
Def. Let P be a simple polygon with t and s on the boundary of P . Let P_L and P_R denote the left and right boundary chain from s to t . P is denoted as a *street*, if P_L and P_R are *weakly visible*, i.e., for any point $p \in P_L$ there is at least one point $q \in P_R$ that is visible, and vice versa. ■



Lower Bound

Theorem No strategy can achieve a path length smaller than $\sqrt{2} \times \pi_{\text{Opt}}$.

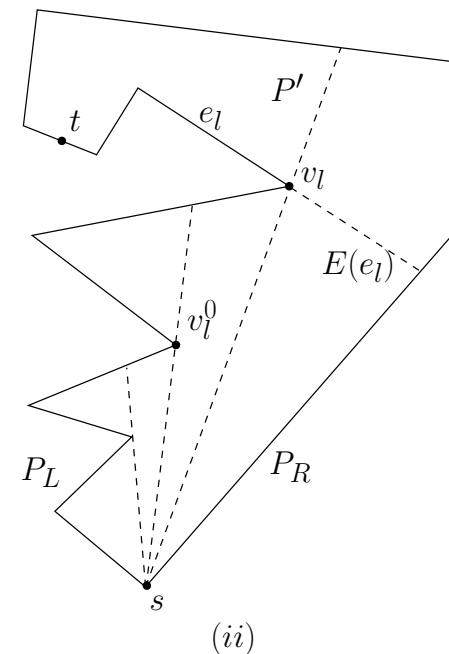
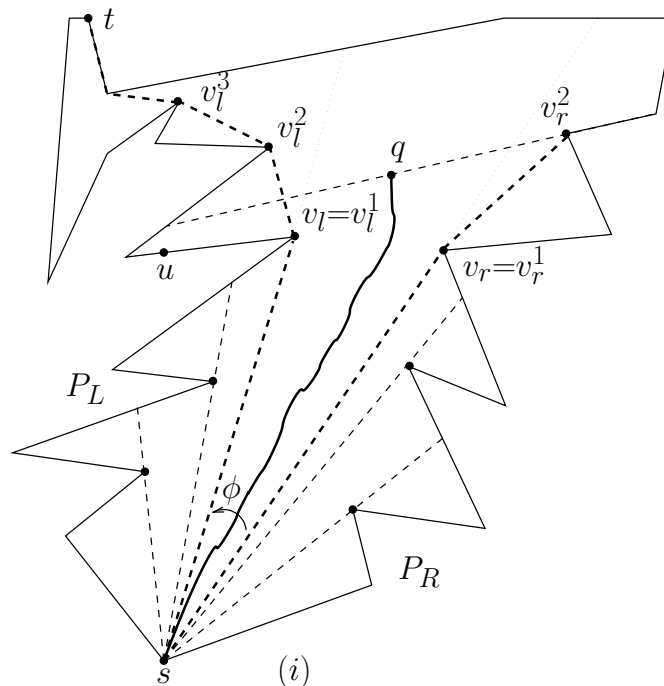
Proof:



Detour with ratio $\sqrt{2}$

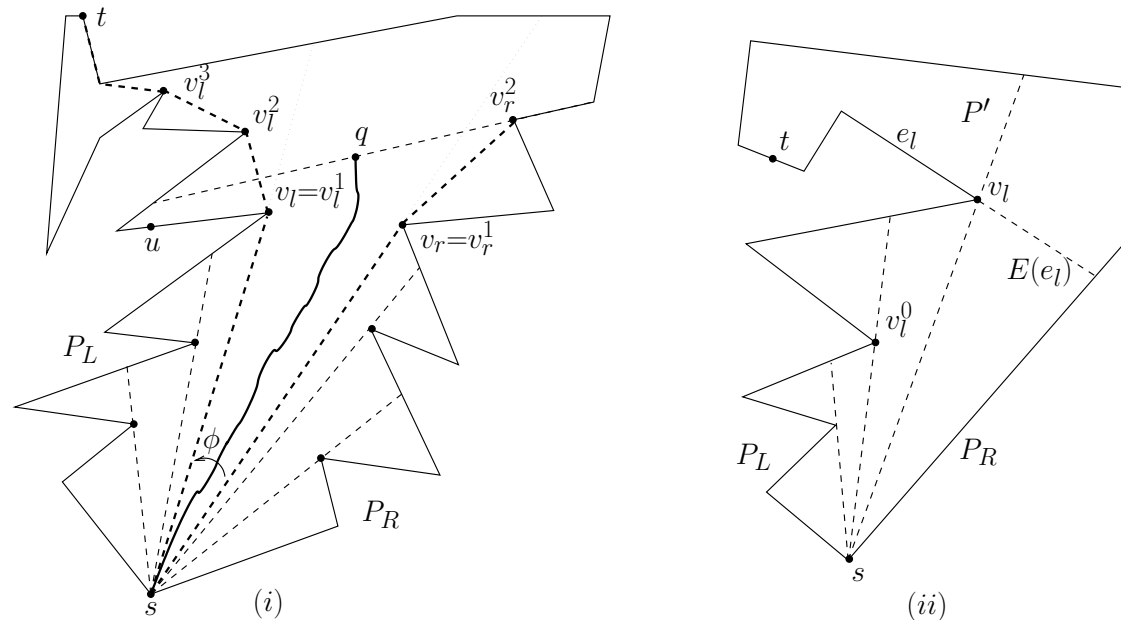
Reasonable movements: Struktural property!

- Inner wedge is important: Between ...
- Rightmost left reflex vertex, leftmost right reflex Vertex
- By contradiction: Assume that the goal is not there! No street!



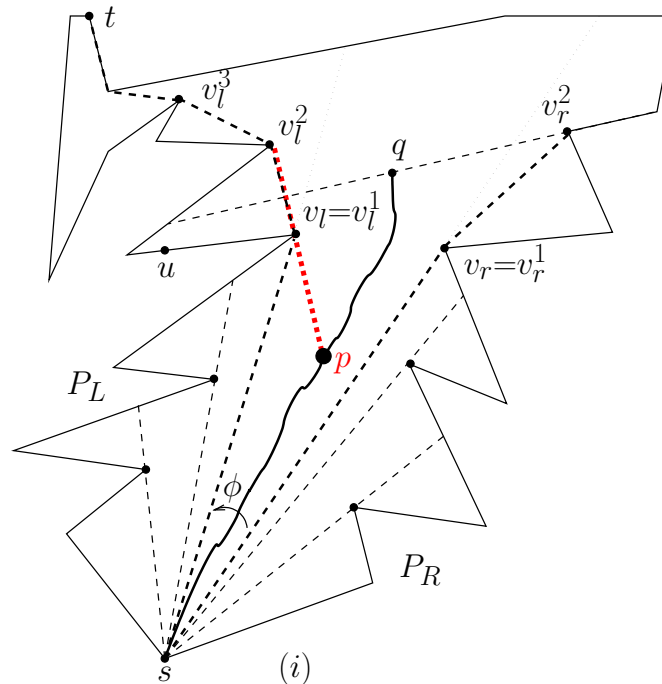
Reasonable strategies

- Always into wedge of c , v_l and v_r ■
- Goal visible, move directly toward it ■
- Cave behind v_l or v_r fully visible, no target as for q (v_l or v_r vanishes), agent moves directly to the opposite vertex ■



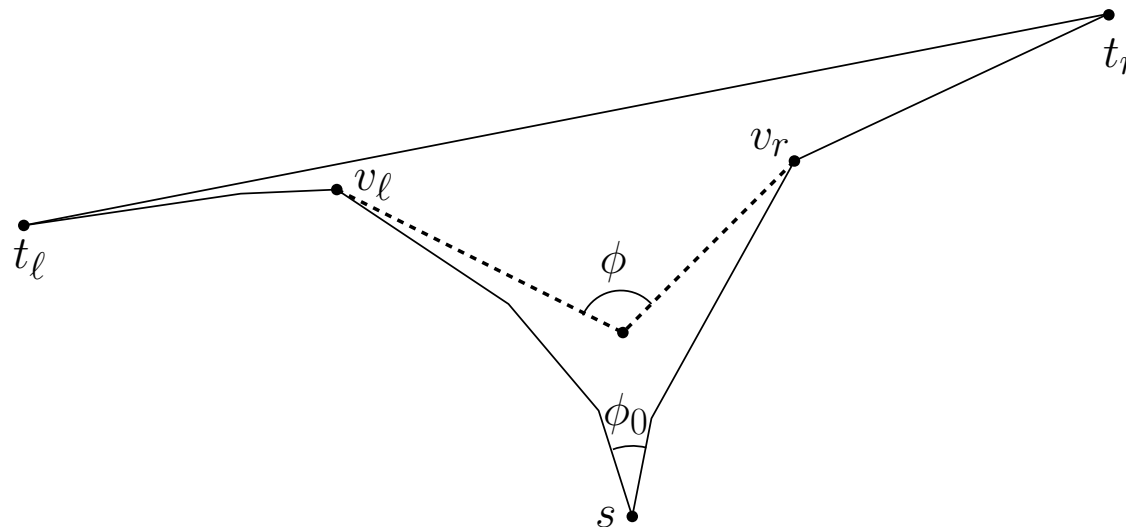
Reasonable strategies

- Always into wedge of c , v_l and v_r ■
- Another vertex (for example) v_l^2 appears behind v_l .
Change to the wedge c , v_l^2 and v_r ■



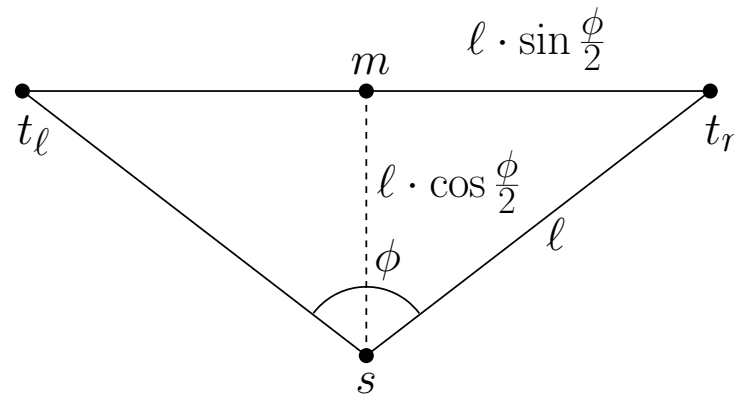
Funnel situation!

- It is sufficient to consider special streets only! ■
- Combine them piecewise!
- **Def.** A polygon that start with a convex vertex s and consists of two opening convex chains ending at t_ℓ and t_r respectively and which are finally connected by a line segment $\overline{t_\ell t_r}$ is called a *funnel* (polygon). ■



Generalized Lower Bound

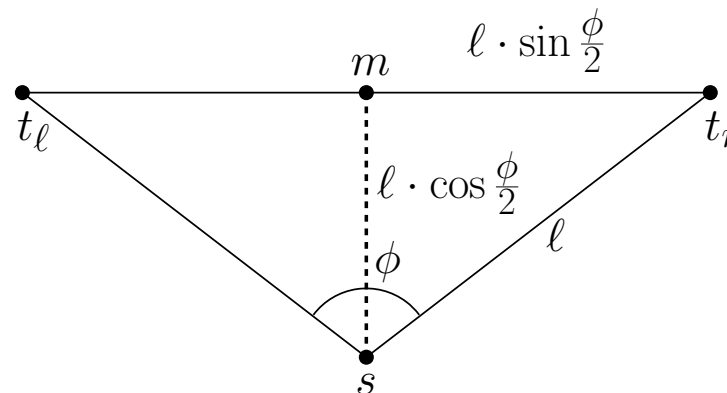
Lemma For a funnel with opening angle $\phi \leq \pi$ no strategy can guarantee a path length smaller than $K_\phi \cdot |Opt|$ where $K_\phi := \sqrt{1 + \sin \phi}$. **Beweis:**



Detour at least: $\frac{|\pi_S|}{|\pi_{Opt}|} = \frac{l \cos \frac{\phi}{2} + l \sin \frac{\phi}{2}}{l} = \sqrt{1 + \sin \phi}$.

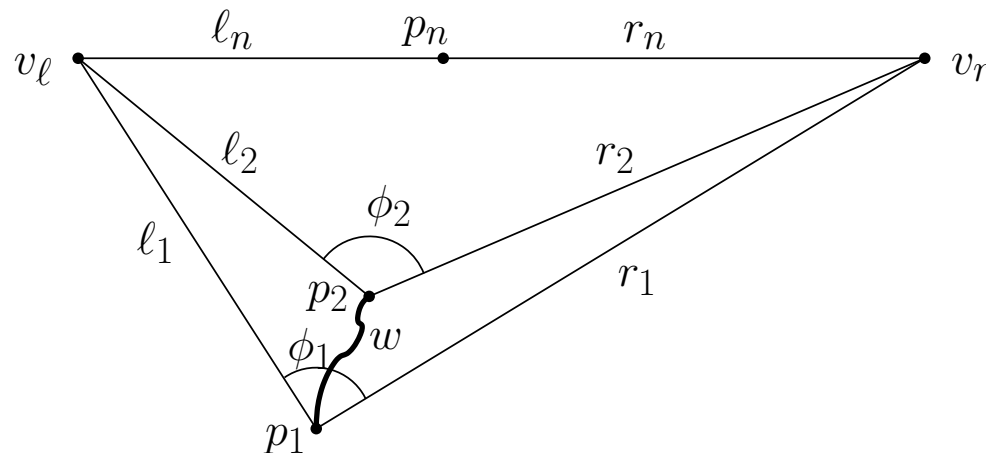
Opt. strat. opening angle $0 \leq \varphi_0 \leq \pi!$

- $K_\phi := \sqrt{1 + \sin \phi}$.■
- Strongly increasing: $0 \leq \phi \leq \pi/2$, Interval $[1, \sqrt{2}]$ ■
- Strongly decreasing: $\pi/2 \leq \phi \leq \pi$, Interval $[\sqrt{2}, 1]$ ■
- Subdivide: Strategy up to $\phi_0 = \pi/2$, Strategy from $\phi_0 = \pi/2$ ■
- Here: Start from s with angle $\phi_0 \geq \pi/2$.■
- Remaining case: Exercise!■



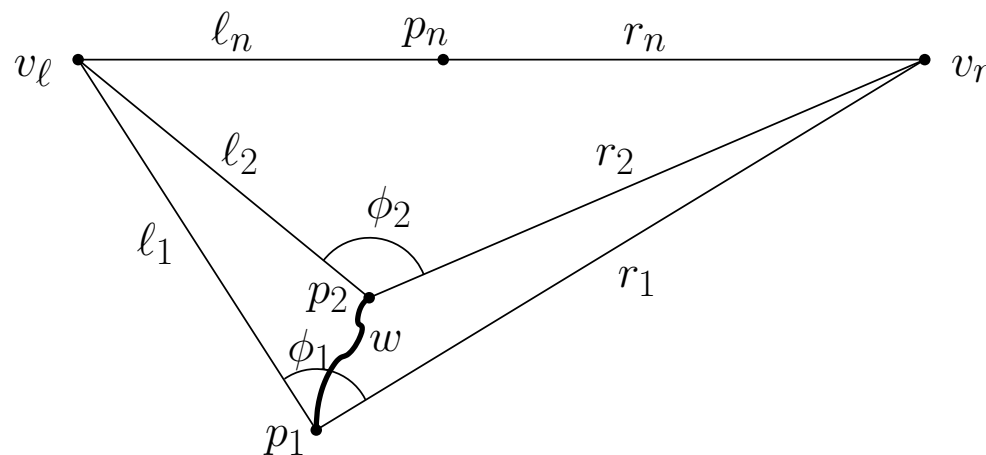
Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2!$

- Backward analysis: For $\varphi_n := \pi$ optimal strategy.■
- $K_\pi = 1$ and K_π -competitive opt. strategy with path l_n or r_n !■
- Assumption: Opt. strategy for some ϕ_2 with factor K_{ϕ_2} ex.■
- How to prolong for ϕ_1 with factor K_{ϕ_1} where $\frac{\pi}{2} \leq \phi_1 < \phi_2$ ■
- We have $K_{\phi_1} > K_{\phi_2}$ ■



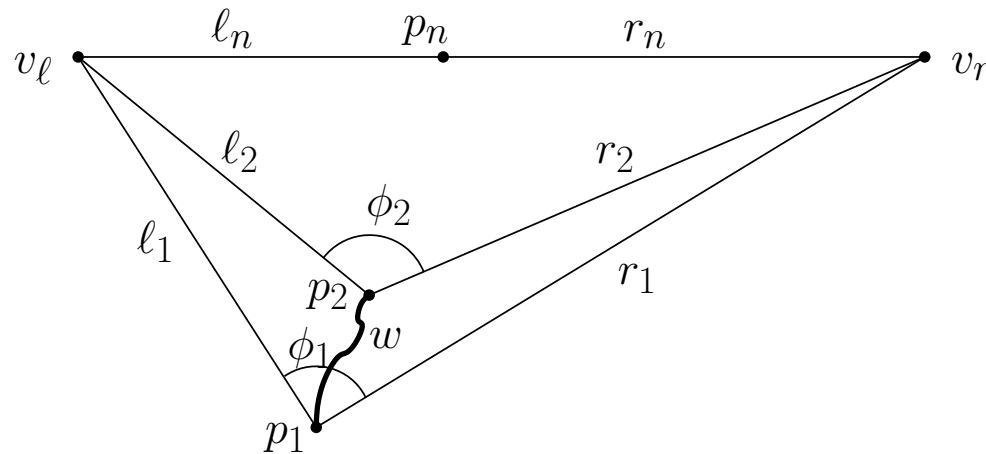
Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2!$

- Situation: Opt. strategy for ϕ_2 with ratio K_{ϕ_2} ■
- How to get opt. strategy for K_{ϕ_1} ? ■
- Conditions for the path w ? Design! ■
- Goal behind v_l , path: $|w| + K_{\phi_2} \cdot l_2$, ■ Optimal: l_1 ■
- Goal behind v_r , path: $|w| + K_{\phi_2} \cdot r_2$, ■ Optimal: r_1 ■
- Means: $\frac{|w| + K_{\phi_2} \cdot l_2}{l_1} \leq K_{\phi_1}$ and $\frac{|w| + K_{\phi_2} \cdot r_2}{r_1} \leq K_{\phi_1}$ ■



Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2!$

- Guarantee: $\frac{|w| + K_{\phi_2} \cdot \ell_2}{l_1} \leq K_{\phi_1}$ and $\frac{|w| + K_{\phi_2} \cdot r_2}{r_1} \leq K_{\phi_1}$ ■
- Combine, single condition for w ■
- $|w| \leq \min\{ K_{\phi_1} l_1 - K_{\phi_2} \ell_2, K_{\phi_1} r_1 - K_{\phi_2} r_2 \}$ ■
- Change of a vertex at p_2 ? ■ Remains guilty! ■



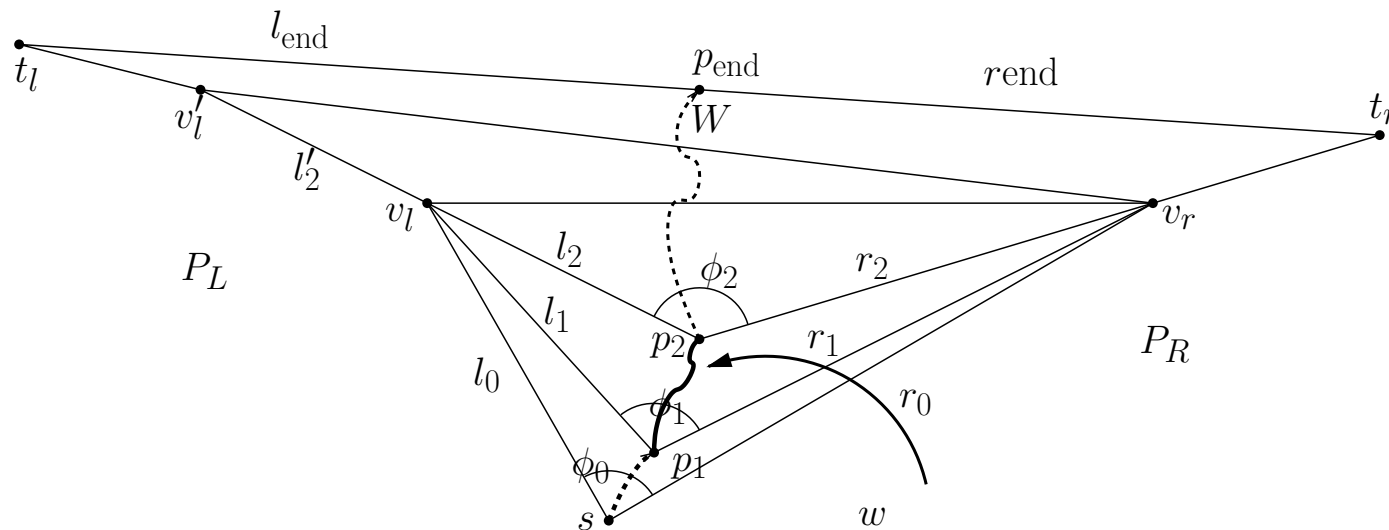
Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2!$

- Change left hand: Condition

$$|w| \leq \min\{ K_{\phi_1} l_1 - K_{\phi_2} l_2, K_{\phi_1} r_1 - K_{\phi_2} r_2 \}$$

- There is opt. strategy for ϕ_2

- Show: $\frac{|w| + K_{\phi_2} \cdot (l_2 + l'_2)}{(l_1 + l'_1)} \leq K_{\phi_1}$

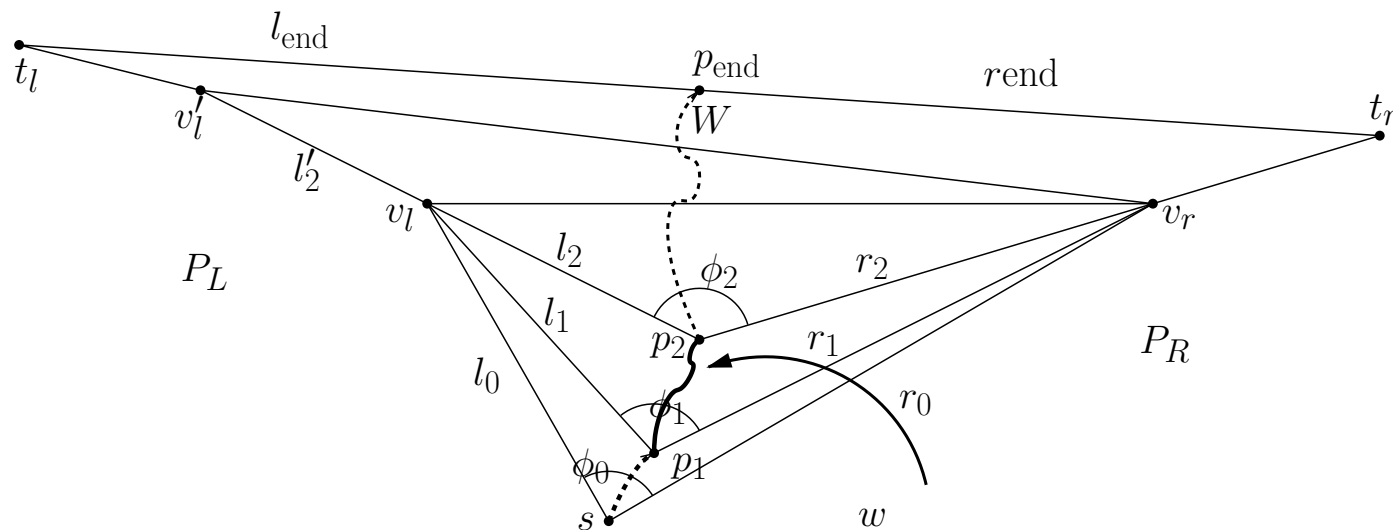


Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2!$

$$|w| \leq K_{\phi_1} l_1 - K_{\phi_2} l_2$$

$$\blacksquare = K_{\phi_1} l_1 - K_{\phi_2} l_2 + K_{\phi_2} l'_2 - K_{\phi_2} l'_2$$

$$\blacksquare \leq K_{\phi_1} (l_1 + l'_2) - K_{\phi_2} (l_2 + l'_2) \blacksquare$$



Opt. strat. opening angle $\pi \geq \varphi_0 \geq \pi/2!$

Lemma Let S be a strategy for funnels with opening angles $\phi_2 \geq \frac{\pi}{2}$ and competitive ratio K_{ϕ_2} . We can extend this strategy to a strategy with ratio K_{ϕ_1} for funnels with opening angles ϕ_1 where $\phi_2 > \phi_1 \geq \frac{\pi}{2}$, if we guarantee

$$|w| \leq \min\{ K_{\phi_1} l_1 - K_{\phi_2} l_2, K_{\phi_1} r_1 - K_{\phi_2} r_2 \}$$

for the path w from p_1 (opening angle ϕ_1) to p_2 (opening angle ϕ_2). ■

