

# Online Motion Planning

## Problem Set 2

Universität Bonn, Institut für Informatik I

*To be solved until the 8th of November*

**Problem 1:**

Prove the following fact: Given two simple grid polygons, we have

$$E(P_1) + E(P_2) = E(P_1 \cup P_2) + E(P_1 \cap P_2).$$

**Problem 2:**

We showed that the  $\ell$ -Offset of a simple grid polygon  $P$  has got  $8\ell$  edges less than  $P$ , if the  $\ell$ -Offset is connected. Generalize this fact to arbitrary simple grid polygons!

**Problem 3:**

In the lecture it was shown that the piecemeal setting can be reduced to the tethered robot setting.

Formulate and prove the correctness of a reduction in the opposite direction. I.e. find a scheme that transforms a given piecemeal algorithm with  $2(1 + \alpha)r$  into a tethered-robot strategy with  $(1 + \beta)r$  and figure out its cost factor.

**Problem 4:**

Prove that an unknown graph with *unknown* radius  $r$  can be explored in  $O(|E| + |V|/\alpha)$  steps by a tethered robot with cable length  $(1 + \alpha)r$ .

**Hint:** Use the modification mentioned in the lecture. You basically will have to repeat the proofs of for the original algorithm. Notice that  $d_{G^*}(s, s_i)$  could be 0. So replace  $d_{G^*}(s, s_i)$  by  $\max(d_{G^*}(s, s_i), c)$  for some constant  $c > 0$  and prove the following instead of Claim 4:

*After every iteration of the main loop in CFX, for every  $T \in \mathcal{T}$ ,*

$$|T| \geq \max(d_{G^*}(s, T), c)\alpha/4$$