Methoden der Offline Bewegungsplanung

Trapezzerlegung, BA-Themen

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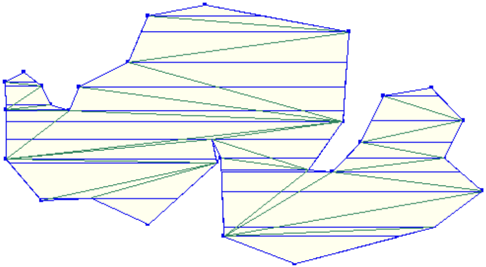
Datenstruktur: Trapezzerlegung nach Seidel

- Lokalisation Black Box
- Polygon: Dreieckszerlegung
- Anfrage: Für $p \in P$ finde Dreieck T mit $p \in T$
- Kürzeste Wege in Polygonen/auf Polyeder
- Aufbau: $O(n \log^* n)$, Query: $O(\log n)$
- Zerlegung in Trapeze, monotone Polygone, danach in Dreiecke
- Hier Trapezzerlegung!



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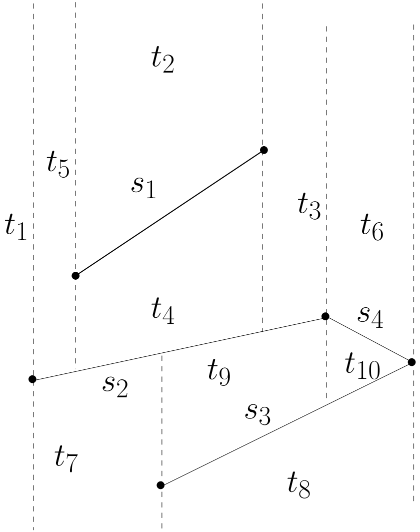

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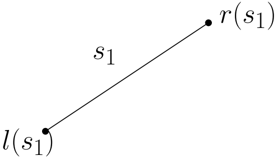

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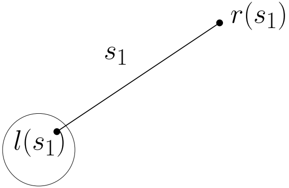

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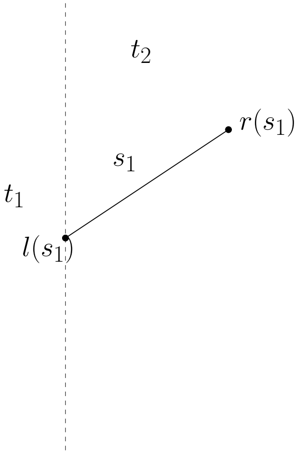

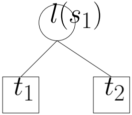

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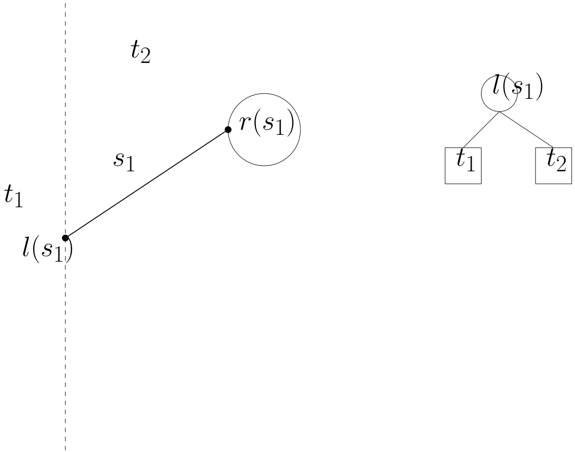

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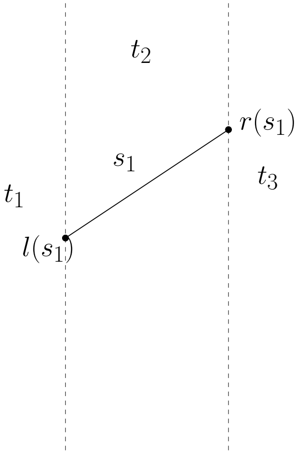

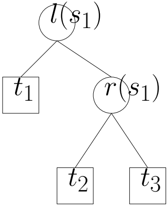

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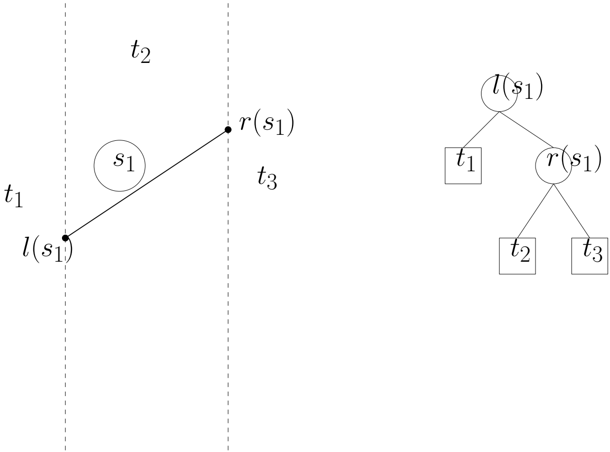

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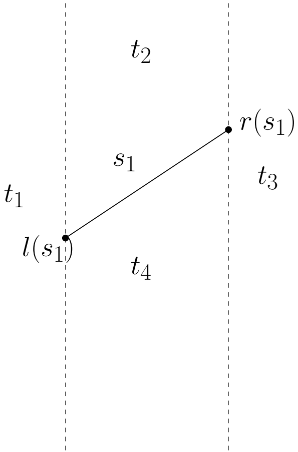

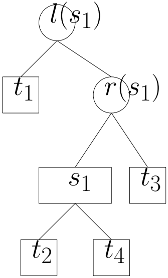

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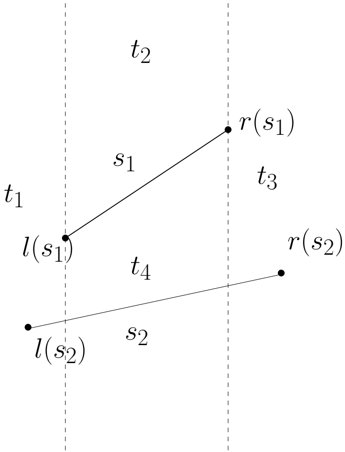

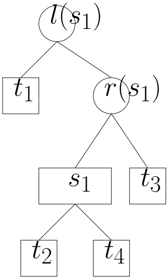

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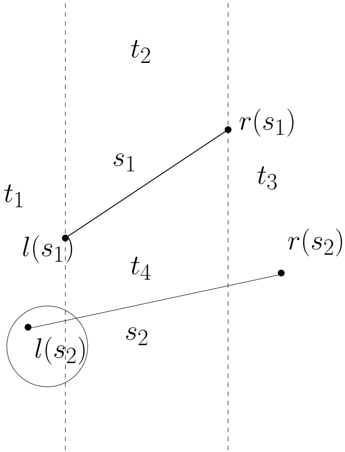

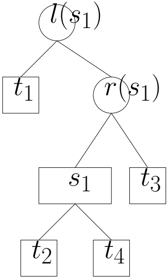

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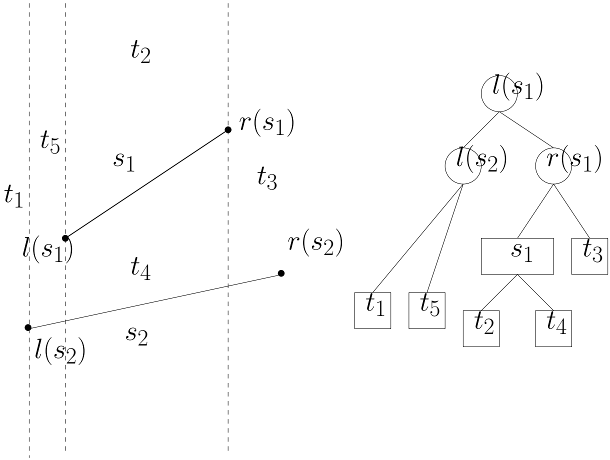

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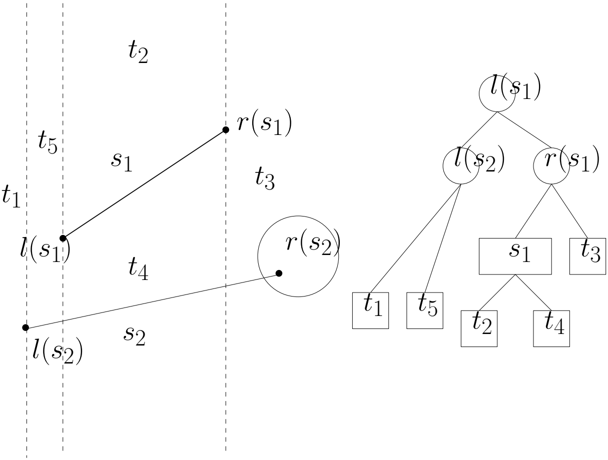

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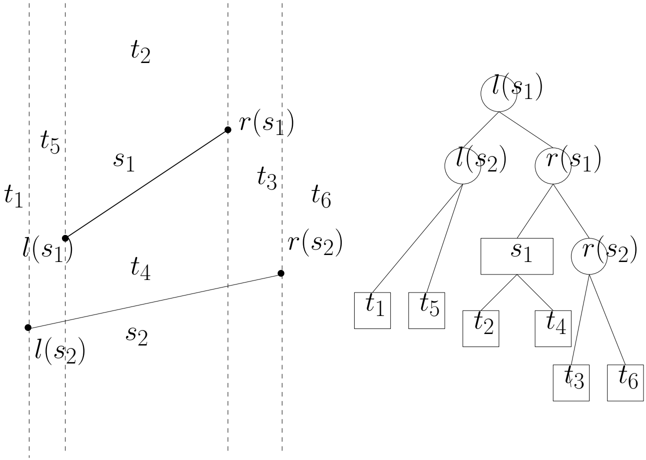

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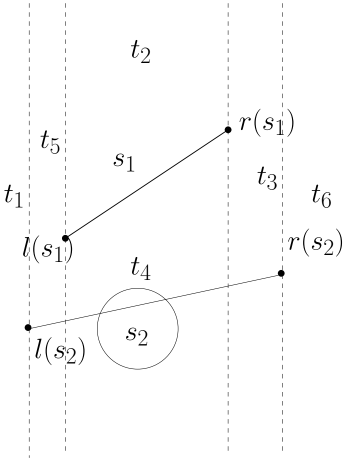

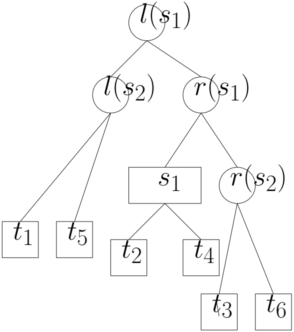

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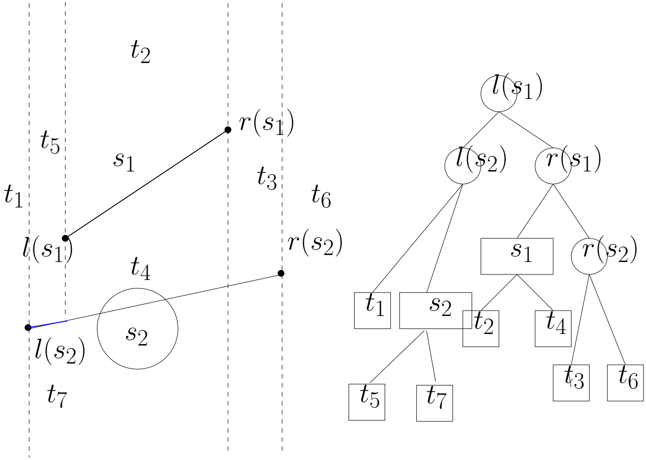

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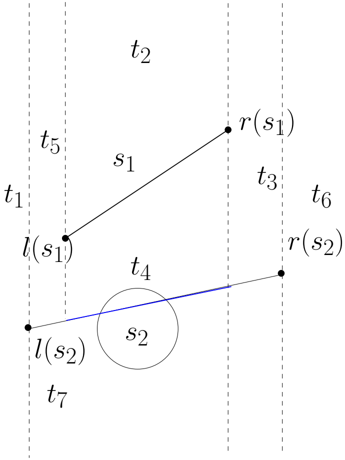

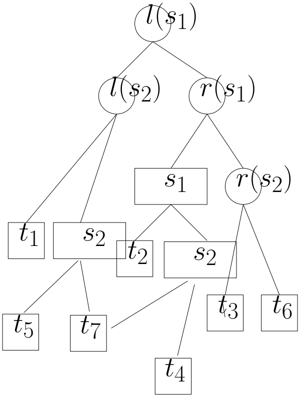

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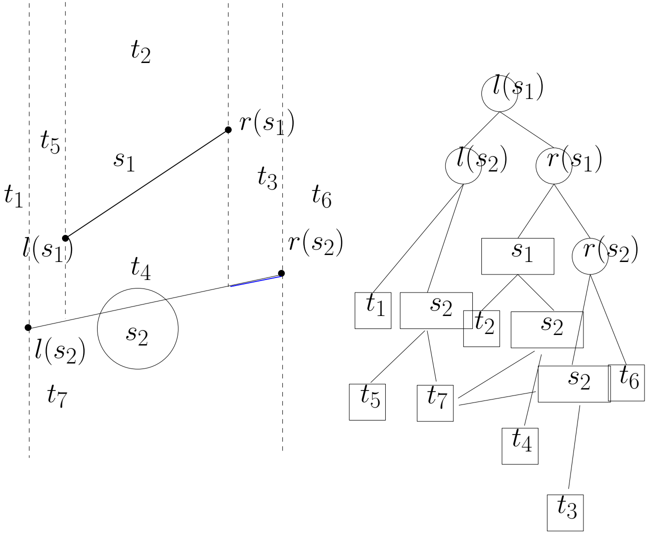

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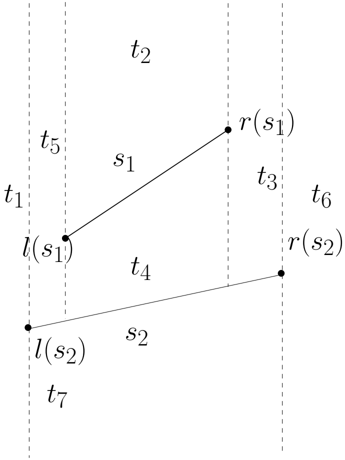

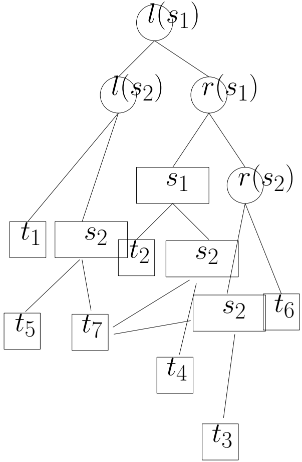

Datenstruktur: Trapezzerlegung nach Seidel

Vorgehensweise:

- Linken Endpunkt lokalisieren
- Rechten Endpunkt lokalisieren
- Strecke s_i durch Trapeze verfolgen
- Trapeze abspeichern mit Nachbarn und Links! $O(1)$
- Wesentlicher Aufwand: Lokalisation
- Query: Lokalisation eines Punktes
- Beweisskizze: Randomisierung! Einfache Analyse!

Datenstruktur: Trapezzerlegung nach Seidel

- Segmente $s_1, s_2, \dots, s_n$ zufällig!
- $P_i = \{s_1, \dots, s_i\}$
- $D(P_i)$ Datenstrucktur nach sukzessiven Einfügen
- $T(P_i)$ (eindeutige) Trapezzerlegung
- Lokalisation der Endpunkte
- Tracen der Segmente, insgesamt $O(n)$ wg. Komplexität

Lemma: Die erwartete Laufzeit bei zufälliger Einfügereihenfolge der Segmente zur Erstellung der Datenstruktur $D(P_n)$ liegt in $O(n \log n)$.

Beweis, Verbesserung durch Phasen, Tracing der polygonalen Kette!

Randomisierte Analyse: Lokalisation der Endpunkte

- Query Punkt q: Sei T_k Trapez von $T(P_k)$, das q enthält
- Betrachte Übergang von $D(P_{j-1})$ nach $D(P_j)$
- Annahme: q liegt in T_{j-1}
- Verändert sich T_{j-1} nicht, gilt $T_{j-1} = T_j$.
  Hier ist kein weiterer Vergleich an dieser Stelle nötig
- Gilt $T_{j-1} \neq T_j$, hat T_j am Rand eines der vier Bestandteile von s_j.
- Jedes Segment hatte die Wahrscheinlichkeit $\frac{4}{j}$, einen Bestandteil zu liefern
- Wahrscheinlichkeit eines weiteren Vergleichs liegt in $O\left(\frac{1}{j}\right)$
- Erwartete Anzahl Vergleiche: $C\sum_{i=1}^{n} \frac{1}{j} \in O(\log n)$

Datenstruktur: Trapezzerlegung nach Seidel

Idee: Schnellere Lokalisation: Polygonale Kette in Phasen in akt. DS lokalisieren.

Theorem: Die erwartete Laufzeit bei zufälliger Einfügereihenfolge der Segmente mit gelegentlicher Lokalisation der polygonalen Kette zur Erstellung der Datenstruktur $D(P_n)$ liegt in $O(n \log^* n)$. Die Zeit pro weiterer Anfrage liegt in $O(\log n)$.

Bewegungsplanung: Feuerbekämpfung

- Wichtiges Problem:
  Schlagen von Feuerschneisen
- Vorhandene Schneisen
  Neue Schneisen
- Einfaches Modell:
  Gleichmäßige Ausbreitung
- Geschwindigkeit: $s \in [0, 1)$

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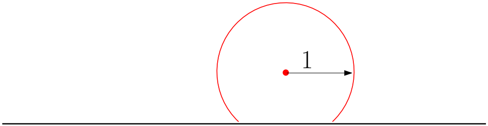

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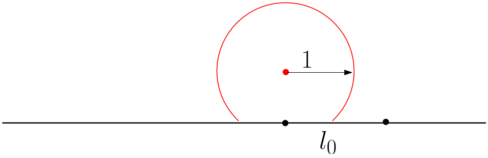

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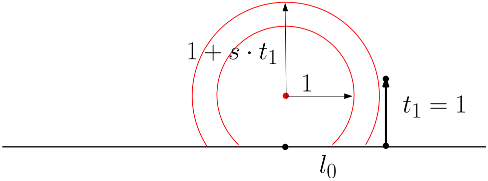

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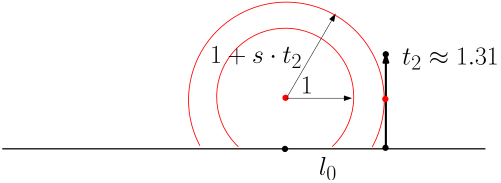

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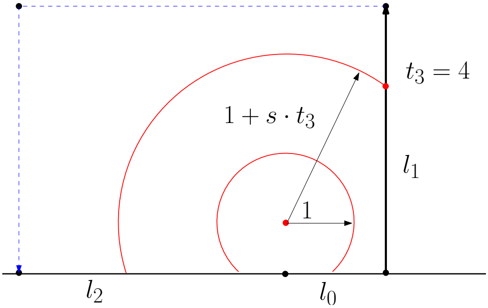

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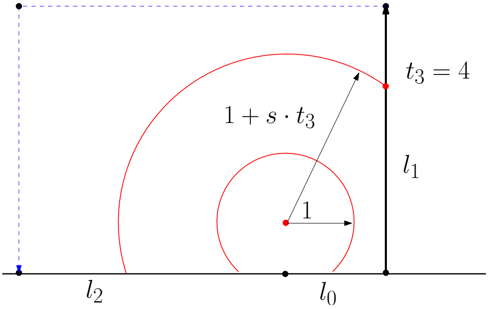

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  umschließen
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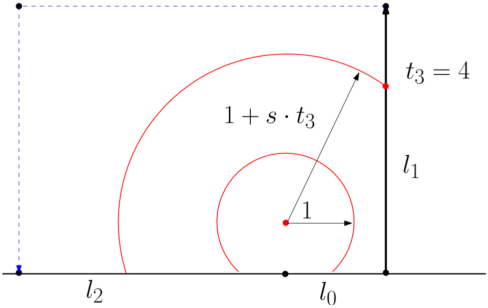

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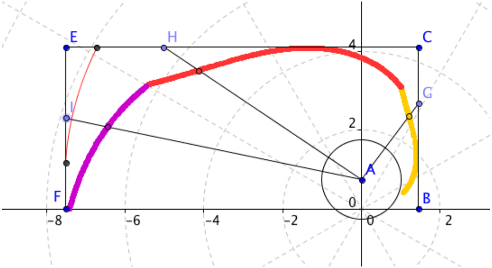
- l_0, l_1, l_2 *gefahrlos*
  minimieren!

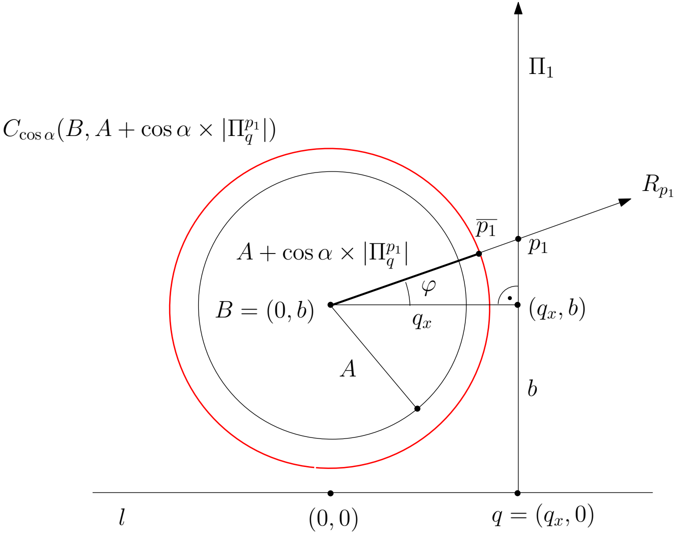


- $s \approx 0.38 = \cos(1.18)$
  Optimale Lösung!
- Minimieren der Werte
- *Passieren* des Feuers
- Zeit *und* Fläche!

Theorem: There exists a unique area and completion time optimal axis-parallel firebreak for all $\alpha \in (\pi/4, \pi/2]$. For $\alpha \in (0, \pi/4]$ no solution exists.

Beweis!

Situation erster Teil!



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- Für $\alpha \in (0, \pi/4]$ keine Lösung!
- Geht das, wenn die Firefigther beliebig graben dürfen?
- Idee: Starte nah am Feuer und bleibe dran!
- Geschwindigkeit cos(α), $\alpha \in (0, \pi/2]$
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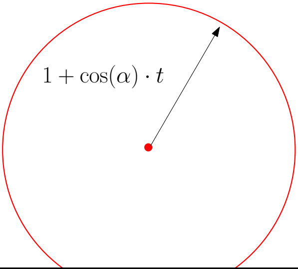

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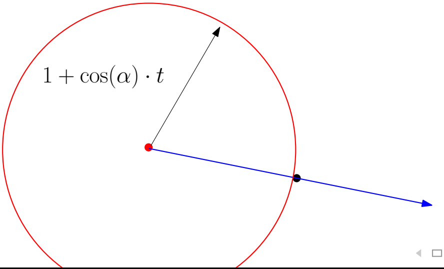

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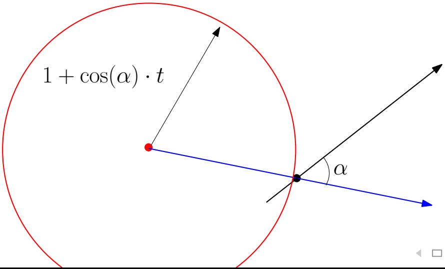

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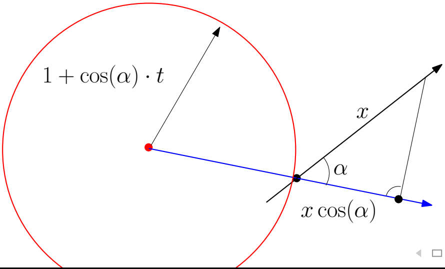

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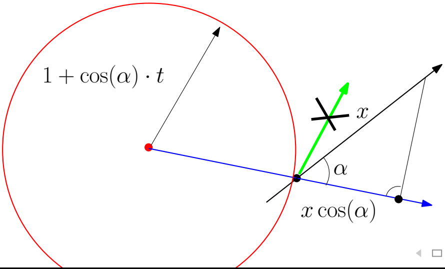

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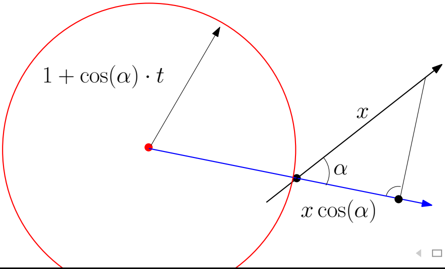

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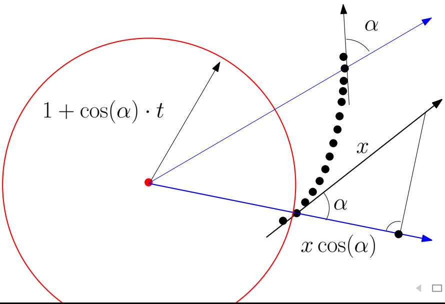

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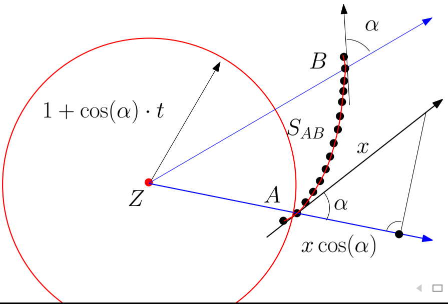
$$\overline{ZA} + \cos(\alpha) \cdot S_{AB} = \overline{ZB}$$



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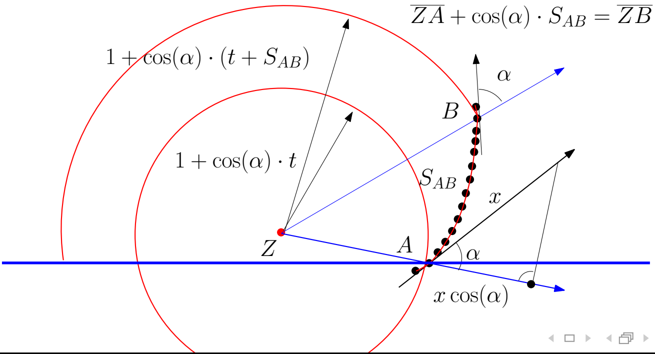

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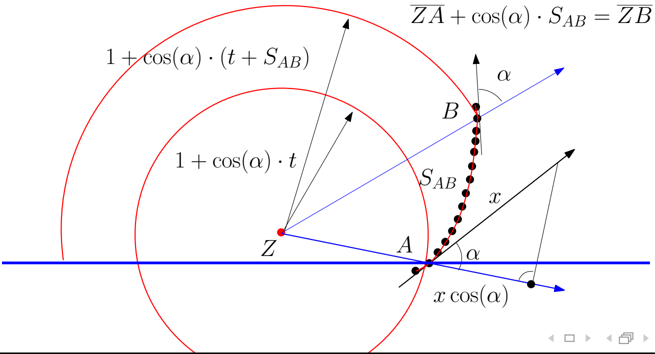

Bewegungsplanung: Feuerbekämpfung

- Mit Spiralen eine bessere/optimale Lösung erstellen!
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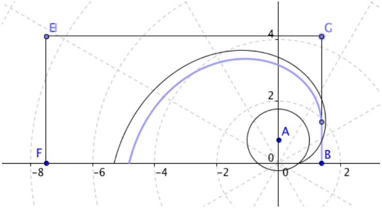

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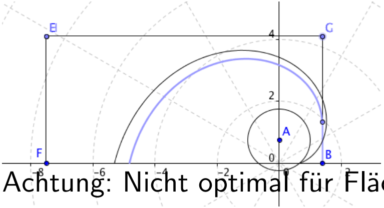
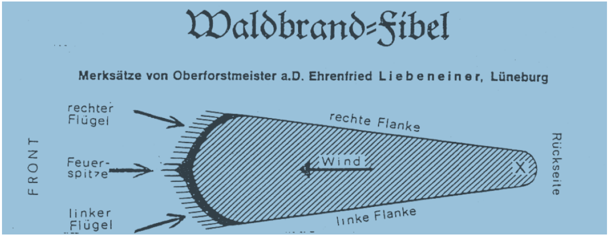


Achtung: Nicht optimal für Flächenminimierung!

Bewegungsplanung: Feuerbekämpfung

- Waldbrände, Grundsätze
- Ausbreitungsgeschwindigkeit klein: 0.5 km/h
- Graben mit Winkel
- Veränderte Ausbreitungskurven
- Modellerweiterungen



Mögliche Aufgabenstellungen

- Kontinuierliches Modell
- Beschränkung der verbrannten Fläche
- Wege mit ein/zwei Agenten, Wege mit wenig Knicken
- Diskretes Modell
- Feuermeldungspfad (auch kontinuierlich)
- Optimale Vorabschneise plus Bekämpfung
- Grid-Graphen oder allgemeine Graphen

Diskrete Problemstellung: Strategic Deployment

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- Modeled by an edge an vertex-weigthed graph

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Example: Minimal number of agents required is 23!

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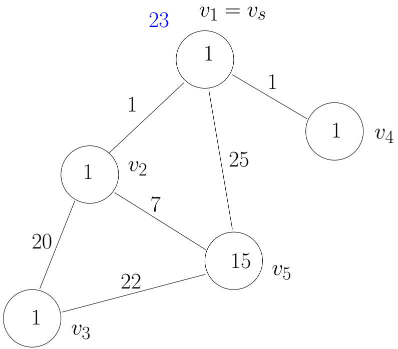

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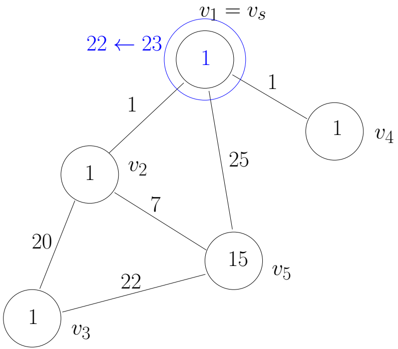

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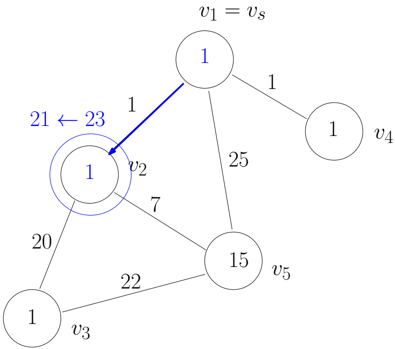

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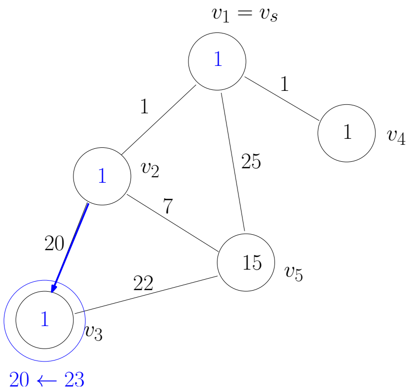

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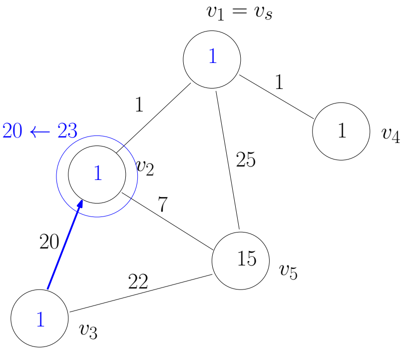

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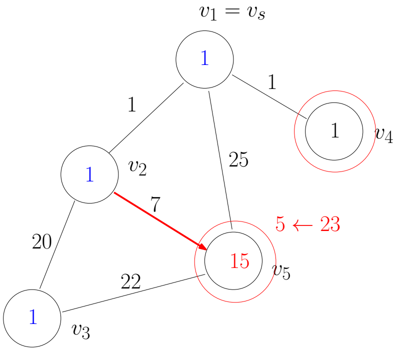

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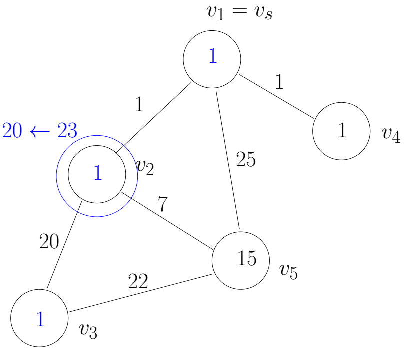

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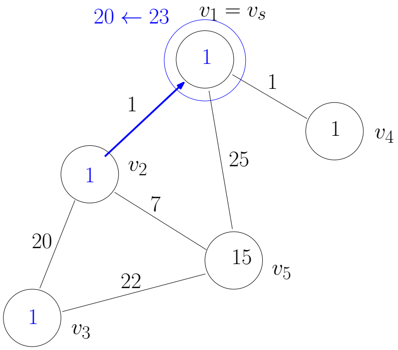

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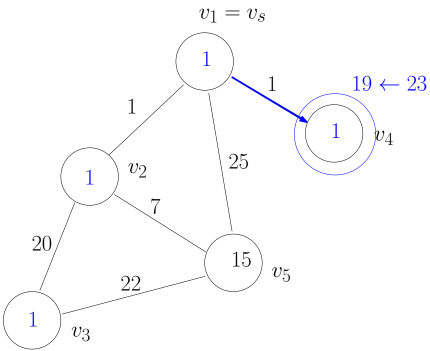

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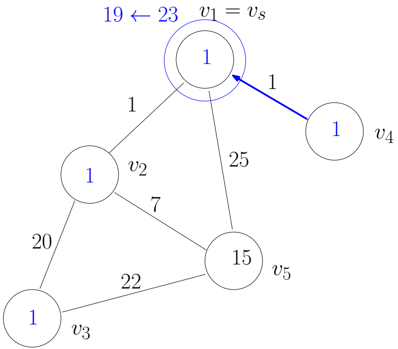

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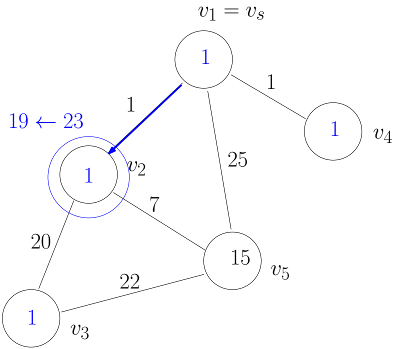

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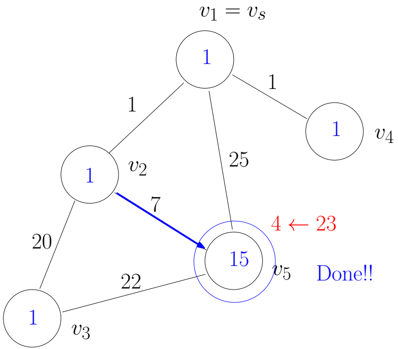

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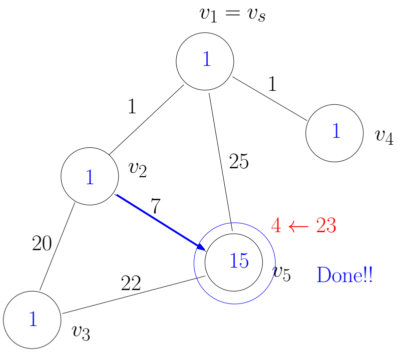

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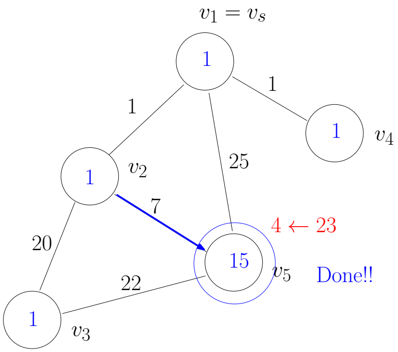

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Is the problem clear?

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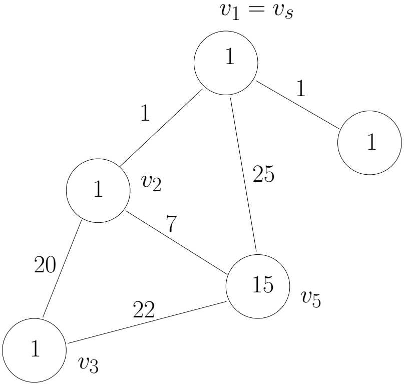

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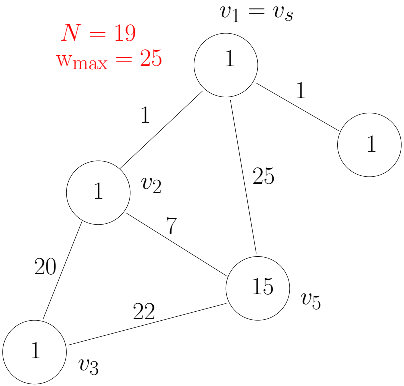

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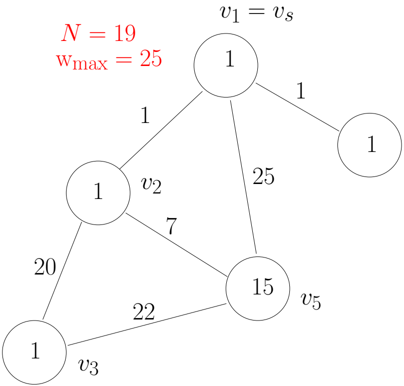

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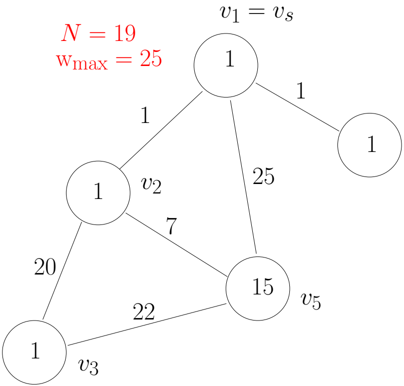

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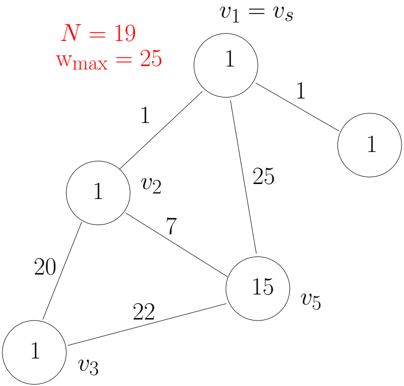

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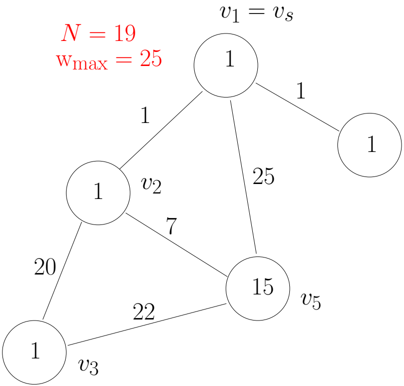

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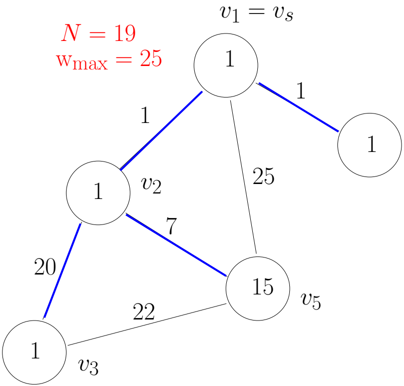

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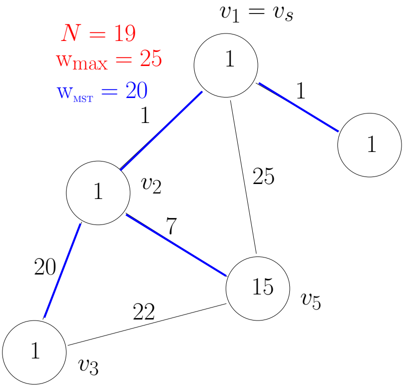

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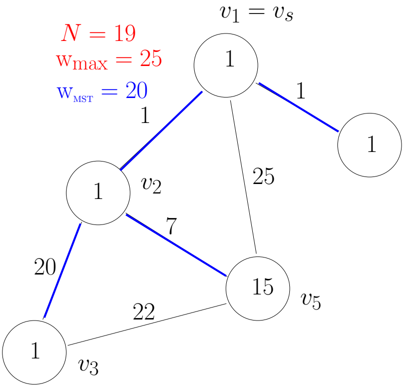

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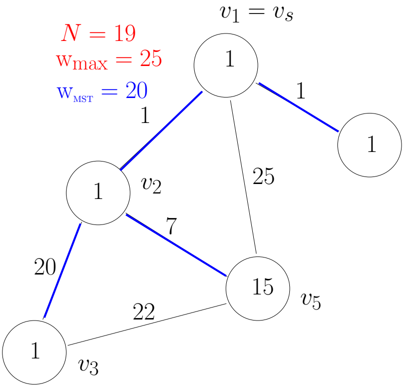

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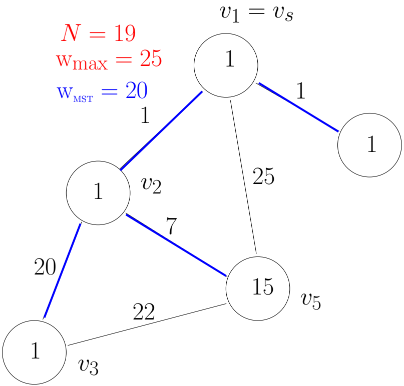
- **Lemma:** Optimal Strategy for MST gives 2-Approximation for G



Variants: Return or No-Return!

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(Return) Finally some agents have to return to the start vertex and report the success of the whole operation.

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Comparable to *routes* (round-trips) and *tours* (open paths) in TSP

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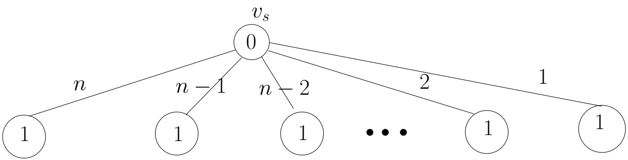
Comparable to *routes* (round-trips) and *tours* (open paths) in TSP

Reporting the success formally means:
Set, M, of agents return to v_s, the union of *all* vertices visited by the members of M equals V.

Optimal Algorithm for Trees: Return Variant

Computational lower bound and algorithmic idea! Example!



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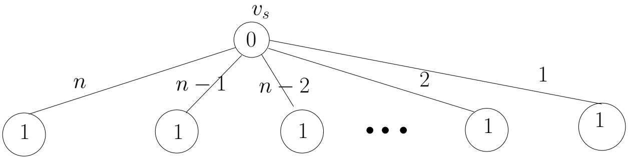

Optimal strategy: $n+1$ agents, visit vertices in order of decreasing edge weights: n,$n-1$,$n-2$,...,2,1

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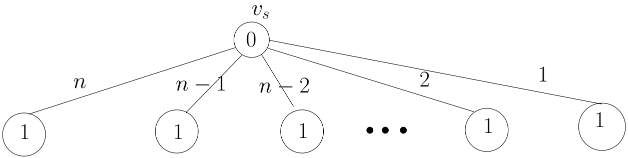

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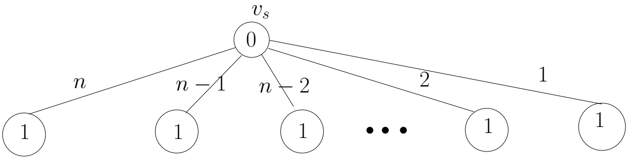

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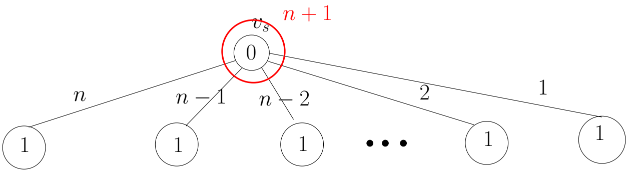

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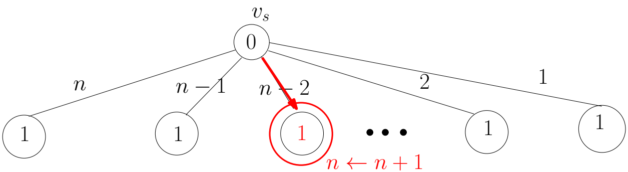

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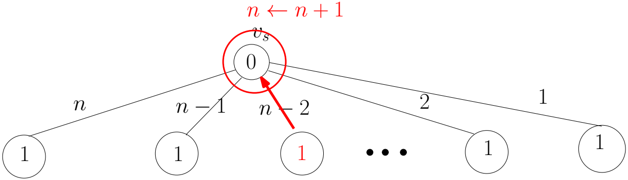

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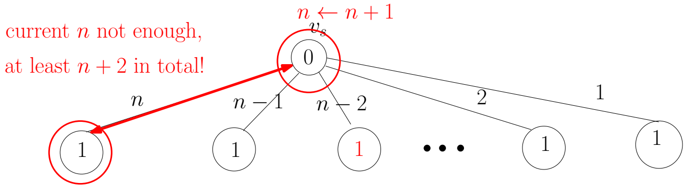

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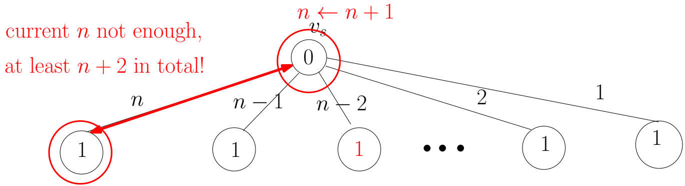

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Lemma: Computational lower bound $\Omega(n \log n)$ by sorting (both variants, but real weights)!

Mögliche Aufgabenstellungen

- Im allgemeinen NP-hard
- Verschiedene Modelle betrachten
- Vollständiger Graph gegeben, gleiche Gewichte, Kantenauswahl?
- Planarer Graph, Euklidische Distanzen als Gewichte, Knotengewicht 1
- Festgelegte Gewichte, Z.B.: 1 oder 2?
- Knoten gesichert, dann Kanten gesichert, freies bewegen, Unterschied?
- Graphen, Bäume