Algorithmic Game Theory
Winter Term 2021/22
Tutorial Session - Week 6

Exercise 1:
Consider the following single-item auction with \( n \geq 2 \) bidders. The bidders simultaneously submit their bids \( b_i \geq 0 \). However, the item will always be allocated to the bidder with index 1 and the mechanism will make him/her pay the bid of the bidder with index 2.

(a) Show that the described mechanism is truthful.

(b) We call a mechanism individually rational if for all bidders \( i \in N \) bidding truthfully against an arbitrary bid profile of the other players never leads to a negative utility: If \( v_i(x) \geq 0 \) for all allocations \( x \in X \), then \( u_i((v_i, b_{\backslash i}), v_i) \geq 0 \).

Show that the given mechanism is not individually rational.

Exercise 2:
Analogous to the auctions that we defined in the lecture, we will consider the following Third-Price Auction. Just like in the first- and second-price auctions, bidders simultaneously submit their bids \( b_i \geq 0 \) and the winner will be determined as the bidder with the highest bid. Finally, the mechanism will make him/her pay the third highest bid. Prove that the described mechanism is not truthful.