

Algorithmic Game Theory and the Internet

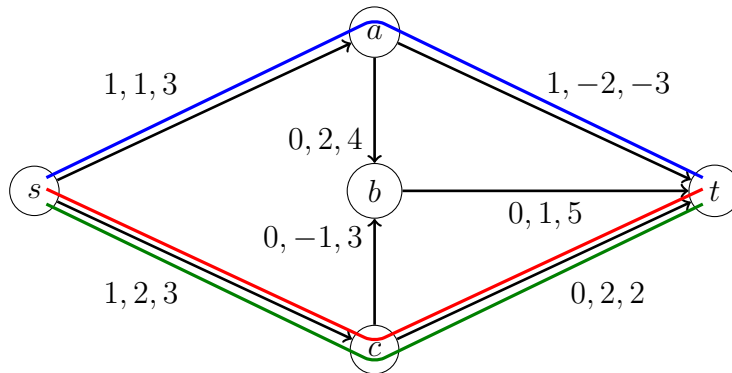
Summer Term 2019

Exercise Set 1

Please hand in your solutions at the beginning of the lecture on 10th of April. You may work in groups of up to three students. Please write down your name(s) on the solutions and tag them as *am* or *pm*, if you are going to participate in the exercises in the morning or afternoon on Thursdays, respectively. The solutions can be written down in German or English.

Exercise 1: (2+4 Points)
 Consider the following symmetric network congestion game with players blue, red and green and their corresponding beginning strategies.

- a) Formalize the network congestion game depicted below. For this purpose, specify the tuple $\Gamma = (\mathcal{N}, \mathcal{R}, (\Sigma_i)_{i \in \mathcal{N}}, (d_r)_{r \in \mathcal{R}})$ like in Example 1.2 of the first lecture. It suffices to state the delay function of a single resource/edge.
- b) Find a pure Nash equilibrium by stating a sequence of best response improvement steps.



Exercise 2: (2 Points)
 Consider congestion games with a constant number of players. Show that the length of every sequence of improvement steps is bounded polynomially in the number of player strategies.

Exercise 3: (6 Points)
 Give an example for a symmetric network congestion game (strategies are s - t paths in a directed graph with the same s and t for all players) with monotonically increasing delay functions d_r such that there exist at least two pure Nash equilibria with different *social costs*. We define the social cost to be the sum of all players' costs $\sum_{i \in \mathcal{N}} c_i(S)$.

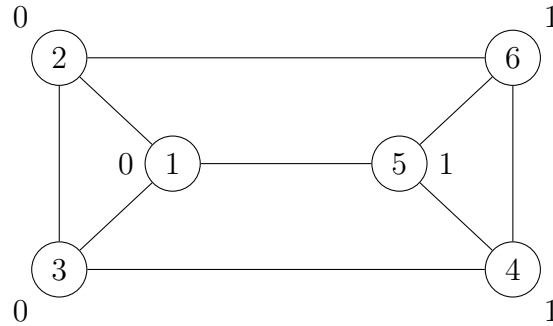
Exercise 4 on the next page.

Exercise 4:

(1+3+2 Points)

In a *consensus game*, we are given an undirected graph $G = (V, E)$ with vertex set $V = \{1, \dots, n\}$. Each vertex $i \in V$ is a player and her action consists of choosing a bit $b_i \in \{0, 1\}$. Let $N(i) = \{j \in V \mid \{i, j\} \in E\}$ denote the set of neighbours of player i , i.e., all players j connected to i via an edge. Furthermore, let $\vec{b} = (b_1, \dots, b_n)$ be the vector of players' choices. The loss $D_i(\vec{b})$ for player i is the number of neighbours that she disagrees with, i.e.,

$$D_i(\vec{b}) = \sum_{j \in N(i)} |b_j - b_i|.$$



- Calculate the loss D_i of player 1 for the player actions depicted in the graph above.
- Show that a consensus game represented as an undirected Graph G can also be modeled as a congestion game Γ . To this end, specify the tuple $\Gamma = (\mathcal{N}, \mathcal{R}, (\Sigma_i)_{i \in \mathcal{N}}, (d_r)_{r \in \mathcal{R}})$ and show that the loss D_i coincides with the cost c_i .
- Prove that in a congestion game modeling a consensus game with $|V| = n$ players all improvement sequences have length $O(n^2)$.