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8th April 2019
Due date: In the lecture on 17th April 2019, 12:15

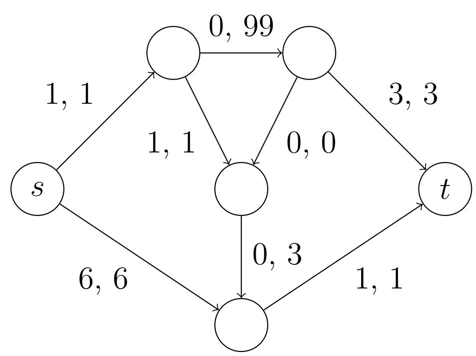
Algorithmic Game Theory and the Internet

Summer Term 2019

Exercise Set 2

Exercise 1: (2 Points)

Rewrite the depicted symmetric network congestion game with two players as a bimatrix game like in Example 3.2 of the third lecture. Why is it sufficient to state only the upper or lower triangular matrix?



Exercise 2: (2+1+3 Points)

a) Specify the payoff matrix for the well-known game rock-paper-scissors[1]. Assume that winning has a cost of -1, losing a cost of 1, a tie a cost of 0.
b) Mark the best responses with boxes. Do we have a pure Nash equilibrium?
c) Determine a mixed Nash equilibrium.

Exercise 3: (3+2 Points)

Consider the bimatrix game *Battle of the Sexes* given in Example 3.3 of the third lecture.

a) Compute a mixed Nash equilibrium by choosing probabilities for one player that will make the other player indifferent between his pure strategies (see Example 3.11).
b) Determine the probabilities of the mixed Nash equilibrium graphically by plotting the players' expected costs.

Exercises 4 and 5 on the next page.

[1] `https://en.wikipedia.org/wiki/Rock%E2%80%93paper%E2%80%93scissors`

Exercise 4: (4 Points)

We define a strategy $s_i \in S_i$ of a normal-form, cost-minimization game to be *strictly dominated*, if there exists a strategy s_i' such that $c_i(s_i', s_{-i}) < c_i(s_i, s_{-i})$ for all $s_{-i} \in S_{-i}$. Prove that for all mixed Nash equilibria σ, there is no player $i \in \mathcal{N}$ with a mixed strategy σ_i such that $\sigma_{i,s_i} > 0$ for a strictly dominated strategy $s_i \in S_i$.

Exercise 5: (3 Points)

Have a look at the proof of Nash's Theorem (4.3) in which normal form payoff-maximization games are considered. Let $\mathcal{N} = \{1, \ldots, n\}$ and $S_i = \{1, \ldots, m_i\}$ for all $i \in \mathcal{N}$. The set of mixed states X can be considered as a subset of $\mathbb{R}^m$ with $m = \sum_{i=1}^{n} m_i$.

Show that X is convex and compact.