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Randomized Algorithms and
Probabilistic Analysis
Summer 2020

Problem Set 1

Please hand in your solutions for this problem set via email (ipsarros@uni-bonn.de) until *Monday, 27th of April.*

Problem 1

We roll two fair standard six-sided dice. What is the probability that

- the number on the first die is larger than the number on the second die.
- the sum of the dice is even.
- the first die is an even number if the sum of the dice is ten.

Problem 2

- Give an example of a probability space and three events A_1, A_2, A_3 such that A_1, A_2, A_3 are pairwise independent, but not independent.
- Let $(\Omega, \Pr)$ be a discrete probability space and let $A, B \in 2^\Omega$ with $\Pr(A) > 0, \Pr(B) > 0$ be events. If A and B are disjoint, can they be independent?

Problem 3

We flip a fair coin n times. We are interested in sequences of tosses that all come up heads. For simplicity, let n be a power of two. Show that the probability that we see a sequence of $1 + \log_2 n$ heads is at most $1/2$. Extend your proof to show that the probability to see a sequence of more than $1 + \log_2 n$ heads decreases exponentially. To do that, show that the probability to see $k + \log_2 n$ heads is upper bounded by $1/2^k$.

Problem 4

A team of three people enters a game show and has to win the following game: Each player gets a hat that he cannot see. The hats are either blue or red, this is decided independently and uniformly at random for each player. Each player can look at the other players hats and then do one of three things:

- Say ‘red’ to guess that his own hat is red.
- Say ‘blue’ to guess that his own hat is blue.
- Say nothing and make no guess.

The players have to answer simultaneously. The team wins if and only if at least one player guesses the color of his own hat correctly and nobody guesses the color of his own hat incorrectly. For example, if two players say nothing and one guesses the color of his own hat correctly, the team wins. As another example, if two players guess correctly and one guesses incorrectly, the team loses.

Describe a strategy for which the team has a probability to win that is at least $1/2$. Then discuss whether it is possible to reach a probability to win of at least $3/4$. If so, describe the strategy.