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Randomized Algorithms and
Probabilistic Analysis
Summer 2020

Problem Set 9

Please hand in your solutions via e-mail (ipsarros@uni-bonn.de) until Monday June 22th.

Problem 1

Suppose that we roll a standard fair dice seventeen times (independently). What is the probability that the sum is divisible by six? Use the principle of deferred decisions.

Problem 2

We want to get more familiar with the notion of ϕ-perturbed values. Assume that c is drawn independently from a distribution with density function $f(x)$, where

1. $f(x)$ is the density of the uniform distribution over the interval $[4, 4+u]$ for a constant $u > 0$.
2. $f(x) = \begin{cases} x^2 \cdot \frac{3}{u^3} & \text{for } x \in [0, u] \\ 0 & \text{else} \end{cases}$ for a constant $u \in (0, \infty)$.
3. $f(x) = \begin{cases} \frac{1}{x} \cdot \frac{1}{\ln u} & \text{for } x \in [1, u] \\ 0 & \text{else} \end{cases}$ for a constant $u \in (1, \infty)$.

For all three cases, do the following:

a) Compute (a best possible) ϕ such that $f(x) \leq \phi$ for all $x \in \mathbb{R}$.

b) Give a best possible upper bound $\nu(u, \epsilon)$ for the probability to draw a number from a given fixed interval of width $\epsilon \in (0, 1)$.

c) Assume someone tells us that c is 3-perturbed. Based on this fixed $\phi = 3$, for which of the three scenarios do we get the largest (i.e. worst) $\nu(u, \epsilon)$?

Problem 3

Let $X_1, \ldots, X_n$ be independent random variables with density functions $f_1, \ldots, f_n$. Furthermore, let $f_i(x) \leq \phi$ for every $i \in \{1, \ldots, n\}$ and every $x \in \mathbb{R}$. Give an upper bound for the probability of $X_1 + X_2 + \ldots + X_n \in [a, a + \varepsilon]$ where $a \in \mathbb{R}$ and $\varepsilon > 0$ are fixed arbitrarily. [Hint: Think of Lemma 6.6!]

Can you extend your reasoning to bound the probability of $\lambda_1 X_1 + \ldots + \lambda_n X_n \in [a, a + \varepsilon]$, where $\lambda_1, \ldots \lambda_n \in \mathbb{R}_+$ are arbitrary numbers?

Problem 4

Let $G = (V, E)$ be a graph with edge lengths $\ell : E \to [0, 1]$ and edge costs $c : E \to [0, 1]$. We are interested in the TSP problem. That is, we assume that G is a complete graph and we want to find a Hamiltonian cycle that is as short and as cheap as possible. Can the set of Pareto-optimal Hamiltonian cycles be computed efficiently (i.e., in polynomial time with respect to the input size and the size of the Pareto set)?