Problem Set 10

Please hand in your solutions via e-mail (ipsarros@uni-bonn.de) until Monday June 29th.

Problem 1

Consider the following *product knapsack problem*. Given n objects with deterministic weights $w_1, \ldots, w_n \in [0, 1]$, a capacity W and deterministic profits $p_1, \ldots, p_n \in \mathbb{R}^{\geq 1}$, find a solution $x \in \{0, 1\}^n$ that maximizes the product

$$p(x) = \prod_{i: x_i = 1} p_i$$

of the profits of the chosen items under the constraint that $w^T x \leq W$. Adapt the Nemhauser-Ullmann algorithm to the product knapsack problem and argue that the adaptation computes an optimal solution to this problem.

Problem 2

Let $G = (V, E)$ be a graph with edge lengths $\ell : E \to [0, 1]$ and edge costs $c : E \to [0, 1]$. Let $\{s, t\} \in V$. We want to find an s-t-path with minimum length as well as minimum total costs. In general there is no such path that optimizes both criteria simultaneously and we are interested in the set of Pareto-optimal paths. Give an algorithm to find the set of Pareto-optimal paths and analyze its worst-case running time.

Problem 3

We agree to try to meet between 12 and 1 for lunch at our favorite sandwich shop. Because of our busy schedules, neither of us is sure when we'll arrive; we assume that, for each of us, our arrival time is uniformly distributed over the hour. So that neither of us has to wait too long, we agree that we will each wait exactly 15 minutes for the other to arrive, and then leave. What is the probability we actually meet each other for lunch?