

Algorithms and Uncertainty

Summer Term 2021

Tutorial Session - Week 3

You are supposed to work on these tasks in class together with your fellow students. Therefore, you are sent into Zoom Breakout-Rooms together with 1-3 other students. Once entered, make sure your camera and microphone are switched on and start with a quick introduction if you do not know each other yet.

Afterwards, you are supposed to discuss the exercises on this sheet. Note that you should see this also as a chance to talk about definitions, proof ideas and techniques in addition to only working out a formal solution for the tasks. If you do not know a definition or theorem by hard, feel free to open the lecture notes and have a look. Further, if you have any questions, I will drop by in your Breakout-Room to discuss possible issues with you.

Exercise 1:

Consider the following rounding algorithm for the Online Set Cover problem. In step t , as a new element e arrives, holding a solution to the fractional set cover problem, we pick all sets $S \in \mathcal{S}$ for which $x_S^{(t)} \geq 1/f$. Again, let $f = \max_{e \in U} |\{S \in \mathcal{S} \mid e \in S\}|$ denote the frequency of the set system, which is known beforehand.

- (a) Show that the rounded integral solution is feasible for the Online Set Cover problem.
- (b) Show that if we use an α -competitive algorithm for the fractional problem, the algorithm for the integral problem is αf -competitive.

Bonus Task:

Our lower bound for Online Set Cover assumes that the algorithm is *lazy*. Show that this assumption is indeed without loss of generality.

Consider an arbitrary online algorithm ALG, which sometimes selects unnecessary or multiple sets. Construct an algorithm ALG' that always selects only a single set and only if it is necessary such that $\text{cost}(\text{ALG}'(\sigma)) \leq \text{cost}(\text{ALG}(\sigma))$ for all σ .

Hint: Keep ALG running in the background.