Thomas Kesselheim
Alexander Braun

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Algorithmic Game Theory

Winter Term 2020/21

Exercise Set 9

Exercise 1: (4 Points)

Consider m items and n bidders. We define a generalization of Walrasian equilibria: Let $S = (S_1, \dots, S_n)$ be an allocation of items to bidders and $q \in \mathbb{R}^m_{\geq 0}$ be a price vector. We call the pair (q, S) an ϵ*-approximate Walrasian equilibrium* if unallocated items have price 0, every bidder i has non-negative utility $v_i(S_i) - \sum_{j \in S_i} q_j \geq 0$, and every bidder receives items within ϵ of its favorite bundle, i.e., $v_i(S_i) - \sum_{j \in S_i} q_j \geq v_i(S'_i) - \sum_{j \in S'_i} q_j - \epsilon$ for every bundle S'_i.

Prove an approximate version of the First Welfare Theorem: If (q, S) is an ϵ-approximate Walrasian equilibrium, then the social welfare of an optimal matching S^* cannot surpass the one of S by more than $\min\{m, n\} \cdot \epsilon$.

Exercise 2: (4+4 Points)

Recall the valuation functions of single-minded bidders from Definition 12.2. Let the maximum bundle size be defined by $d = \max_{i \in \mathcal{N}} |S^*_i|$.

(a) Show that in the case of single-minded bidders with maximum bundle size d, item bidding with first price payments is $(\frac{1}{2}, 2d)$-smooth.

Hint: In order to define deviation bids $b^*_{i,j}$, consider a welfare-maximization allocation on v. If bidder i does not get his bundle in the optimal allocation, then define $b^*_{i,j} = 0$ for all items $j \in M$. Otherwise, define $b^*_{i,j} = \frac{v_i}{2d}$ for all $j \in S^*_i$ and $b^*_{i,j} = 0$ if $j \notin S^*_i$. That is, each winner in the optimal allocation equally divides the value for his bundle among all items of the bundle and bids half of it.

(b) Now, we define prices for items by setting

$$p^v_j = \begin{cases} \frac{1}{2d} v_i(S^*_i) & \text{if buyer } i \text{ gets item } j \text{ in optimal solution on } v \\ 0 & \text{if item } j \text{ is unassigned in optimal solution on } v \end{cases}$$

Be inspired from Step 1 in Lecture 18 to show that using these prices in the full-information setting implies a $\frac{1}{2d}$-approximation of the optimal social welfare.