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Algorithmic Game Theory

Winter Term 2020/21

Tutorial Session - Week 6

Exercise 1:
We would like to prove an alternative characterization of truthful mechanisms which is stated in the following theorem. Moreover, this restated characterization holds for arbitrary mechanisms and not only for single-parameter mechanisms.

Theorem. A mechanism $\mathcal{M} = (f, p)$ is truthful if and only if the following two conditions are met.

(i) For every pair b_i, b'_i: If $f(b_i, b_{-i}) = f(b'_i, b_{-i})$, then we also have $p_i(b_i, b_{-i}) = p_i(b'_i, b_{-i})$. In other words: For all b_{-i}, for all $a \in X$ there exist prices $p_a \in \mathbb{R}$ such that for all b_i with $f(b_i, b_{-i}) = a$ we have $p_i(b_i, b_{-i}) = p_a$.

(ii) The mechanism optimizes for each player. Formally: For every pair b_i, b_{-i} the following holds
$$f(b_i, b_{-i}) \in \arg\max_{a \in A}(b_i(a) - p_a),$$
where the set of allocations A is equal to the image of $f(\cdot, b_{-i})$.

For this purpose, prove the following claims:

(a) If condition (i) is violated, then the mechanism $\mathcal{M}$ cannot be truthful.

(b) If condition (ii) is violated, then the mechanism $\mathcal{M}$ cannot be truthful.

(c) If conditions (i) and (ii) are met, then $\mathcal{M}$ is a truthful mechanism.

Hint: You should not use Myerson's Lemma in this task (since it is not helpful). Instead, it is sufficient to make use of the truthfulness inequality with reasonably chosen deviations. See also how we derived *payment difference sandwich* in the proof of Theorem 11.2.