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Randomized Algorithms and
Probabilistic Analysis
Summer 2017

Problem Set 8

Problem 1

Suppose that we roll a standard fair dice seventeen times (independently). What is the probability that the sum is divisible by six? Use the principle of deferred decisions.

Problem 2

The Nemhauser-Ullmann algorithm is not the algorithm we usually learn for the knapsack problem in Bachelor algorithms courses. Indeed, the knapsack problem can be solved by an algorithm with running time $\mathcal{O}(nW)$, where we assume that the input consists of n items with deterministic integral profits $p_1, \ldots, p_n \geq 0$, deterministic integral weights $w_1, \ldots, w_n \geq 0$ and a capacity $W \leq \sum_{i=1} w_i$. Briefly describe/explain an algorithm that achieves this. What does its existence mean for the complexity of the knapsack problem?

Let's familiarize ourselves a bit with continuous probability spaces, which are defined and discussed in Section 5 of the lecture notes. In particular, we look at the uniform distribution (see page 85). Also notice that independent events are defined as before (see Definition 5.11), and that Lemma 2.10 generalizes to Lemma 5.8.

Problem 3

Assume that we have a continuous roulette wheel. We spin it and then it halts at a position P between $[0,1]$. The position is chosen uniformly at random, i.e., P is uniformly distributed on $[0,1]$.

1. What is the probability to get the number 0.2514?
2. What is $\mathbf{E}(P)$? More generally, let Q be uniformly distributed over $[a,b]$ and compute $\mathbf{E}(Q)$.

Now we spin the wheel twice. Let Y_1, Y_2 be the outcomes. We assume that Y_1 and Y_2 are independent, both uniformly distributed on $[0,1]$. Let $Y_{\min} := \min\{Y_1, Y_2\}$ be the smaller number of the two runs.

3. What is the probability that $Y_{\min} \geq y$ for an $y \in [0,1]$?
4. What is the probability that $Y_{\min} \geq y$? for an $y \in (1,\infty]$?
5. Compute $\mathbf{E}(Y_{\min})$.
6. Compute $\mathbf{E}(Y_{\max})$, where $Y_{\max} = \max\{Y_1, Y_2\}$.