# Problem Set 6

Please hand in your solutions for this problem set via email (roesner@cs.uni-bonn.de) or personally at Room 2.060 until *Tuesday, 20th of November.*

## Problem 1
Let us look at instances of the fair $k$-center problem. Show that the factor between the value of the optimal fair solution and the optimal unfair solution can be unbounded.

## Problem 2
What changes when we instead of calling **farthest-first-traversal**$((P^0, d), k)$ in step 3 of the **fair-k-center** algorithm, we call **farthest-first-traversal**$((P, d), k)$ to compute $C$?

## Problem 3
We would like to extend the fair $k$-center problem to more general settings. Specifically we would like to replace the restriction that the clusters are $(\frac{1}{t}, 1)$-balanced for some $t \in \mathbb{N}$. Instead we would prefer to require that the clusters are $(\ell, u)$-balanced for arbitrary $\ell \leq u \in \mathbb{Q}$. Unfortunately we will see that this scenario seems to be more difficult.

- Assume that we know how the optimal fair clustering clusters the points in $P^0$. Show that we can then compute a 3-approximation.

One approach could now be to first compute an approximate unfair solution on $P^0$ and then try to add the points from $P^1$.

- Show specifically for the case $\ell = ratio(P, 0) = u$ that there are instances where it is impossible to make such a clustering obtained on $P^0$ fair by adding the points from $P^1$.

We want to keep the focus on the exact case where we have $\ell = ratio(P, 0) = u$. Assume that $ratio(P, 0) = \frac{a}{a+b}$ for some coprime integers $a \leq b$.

- Show that in every fair cluster the number of points from $P^0$ must be an integer multiple of $a$.

For the next task we assume that we know an approximation algorithm for the capacitated $k$-center problem. The capacitated $k$-center problem is in addition to $P$, $d$ and $k$ given a capacity $cap$ and demands that each cluster contains at most $cap$ points.

- Show how to compute a clustering on $P^0$, where the number of points in each cluster is an integer multiple of $a$ and whose maximal radius is in $O(opt)$, where $opt$ denotes the value of the optimal fair clustering.

- Use this clustering on $P^0$ to compute a fair clustering. What approximation factor do you obtain?